المعهد العالي للهندسة والتكنولوجيا الحديثة - بالمرج
Higher Institute of Engineering and Modern Technology – El Marg

ELECTRICAL MEASUREMENTS & TESTS

From References

1- **David A. Bell, "Electronic Instrumentation And Measurements", second edition, 2003.**
2- **Larry D. Jones, A. Foster Chin, "Electronic Instruments and Measurements", second edition, 1983.**

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Chapter 1

Introduction To Instrumentation

1-1 Instructional Objectives

This introductory chapter covers electrical units, the different classes of standards, the functions and characteristics of instruments, and various types of errors along with methods for error analysis.

1-2 INRODUCTION

Over the past 150 years, significant advancements have been made in measuring electrical quantities. For much of this time, the focus was on improving deflection-type instruments, which feature a scale and a movable pointer. The pointer’s angle of deflection reflects the value of the electrical quantity being measured.

The term "analog instrument" was introduced to describe these deflection-type instruments and to differentiate them from a different category of devices that show measurement values in decimal (digital) form. These newer devices are referred to as digital instruments.

1-3 FUNCTIONS AND CHARACTERISTICS of INSTRUMENTS

Many instruments are designed to provide information about a variable quantity that needs to be measured. In addition to offering a visual indication of the measured value, some instruments also produce a permanent record of the measurement. Furthermore, certain instruments are used to control a quantity. Thus, instruments generally serve three primary functions: indicating, recording, and controlling. An instrument may perform any one of these functions, or a combination of them, simultaneously. General-purpose electrical and electronic test instruments mainly focus on indicating and recording functions. In industrial-process settings, instrumentation often includes a control function, contributing to what is known as a control or automated system.

There are various methods for measuring different quantities. Some physical quantities are best measured mechanically, such as using a manometer gauge to assess gas pressure. Others are measured through electrical methods, like determining solution conductivity with a current meter. Electronic instruments, which include amplifying circuits to enhance the amplitude of the measured quantity, are also used for certain measurements.

1-4 Electrical units

Since electrical and electronic instruments measure electrical quantities, this introductory chapter will cover electrical units. As with any field of quantitative science, a standardized system of units is essential for making precise measurements and evaluations.

There are six electrical quantities that are relevant for electrical measurements. To analyze these quantities quantitatively, we need to establish and define a system of units. These quantities are: (1) electric charge (Q), (2) electric current (I), (3) electromotive force or potential difference (V), (4) resistance (R), (5) inductance (L), and (6) capacitance (C). Table 1-1 lists the fundamental quantities of the SI system of units, along with some electrical quantities that can be derived from these fundamental units.

Table 1-1

Six Fundamental and Some Derived Quantities of the SI System

Quantity	Symbol	Unit	Unit Abbreviation	Dimensions
Fundamental				
Length	l	meter	m	l
Mass	m	kilogram	kg	m
Time	t	second	s	t
Temperature	T	degree Kelvin	°K	T
Luminous intensity		candela	cd	
Electric current	I	ampere	A	i
Derived				
Electromotive force	V	volt	V	$l^2mt^{-3}i^{-1}$
Quantity of charge	Q	coulomb	C	it
Electrical resistance	R	ohm	Ω	$ml^2t^{-2}i^{-2}$
Capacitance	C	farad	F	$l^{-2}m^{-1}t^4i^2$
Inductance	L	henry	H	$l^2mt^{-2}i^{-2}$

1-5 ERROR IN MEASUREMENT

Measurement involves comparing an unknown quantity to a standard quantity. This process requires connecting a measuring instrument to the system being examined and observing the response on the instrument. The measurement obtained is intended to represent the true value, although defining the true value precisely is challenging. Therefore, the term "expected value" is used instead. Measurements are influenced by various factors, and as a result, the outcomes often deviate from the expected value. For instance, connecting a measuring instrument to a circuit can alter the circuit, leading to discrepancies between the measurement and the expected value.

Factors affecting measurements include the characteristics of the instruments and the user’s actions. The accuracy of a measurement in relation to the expected value is referred to as measurement error, which can be expressed as either

absolute error or percent error. Absolute error is defined as the difference between the expected value and the measured value.

$$e = Y_n - X_n \quad (1\text{-}1)$$

where

e= absolute error

Yn = expected value

Xn = measured value

To express the error as a percentage, we can calculate it as follows.

$$\text{Percent error} = \frac{\text{Absolute error}}{\text{Expected value}}(100) \quad (1\text{-}2)$$

or

$$\text{Percent error} = \frac{e}{Y_n}(100) \quad (1\text{-}3)$$

Substituting for e yields

$$\text{Percent error} = \left|\frac{Y_n - X_n}{Y_n}\right|(100) \quad (1\text{-}4)$$

It is often preferable to express measurements in terms of relative accuracy rather than focusing solely on error.

$$A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right| \quad (1\text{-}5)$$

where A is the relative accuracy.

Accuracy can be expressed as percent accuracy, a. is

a = 100% - Percent error=A x 100 (1-6)

Example 1-1

Given that the expected voltage across a resistor is 50 V and the measured voltage is 49 V, calculate the error.

a) The absolute error.
b) The percent of error.
c) The relative accuracy.
d) The percent of accuracy.

(a) $e = Y_n - X_n$

$= 50\ \text{V} - 49\ \text{V} = 1\ \text{V}$

(b) $\text{Percent error} = \dfrac{50\ \text{V} - 49\ \text{V}}{50\ \text{V}} \times 100\%$

$= \dfrac{1}{50} \times 100\% = 2\%$

(c) $A = 1 - \dfrac{50\ \text{V} - 49\ \text{V}}{50\ \text{V}} = 1 - \dfrac{1}{50}$

$= 1 - 0.2 = 0.98$

(d) $a = 100\% - \text{Percent error} = 100\% - 2\% = 98\%$

$= A \times 100 = 0.98 \times 100 = 98\%$

If a measurement is accurate, it must also be precise; in other words, accuracy requires precision. However, the reverse is not necessarily true—precision does not always ensure accuracy. Precision indicates how consistently repeated measurements of the same variable cluster around their average value and can be described mathematically or numerically.

$$\text{Precision} = 1 - \left| \frac{X_n - \bar{X}_n}{\bar{X}_n} \right| \qquad (1\text{-}7)$$

where

X_n = the value of the nth measurement
$\bar{X}_n$ = the average of the set of n measurements

Example 1-2

The laboratory recorded the following set of ten measurements. Calculate the precision for the fourth measurement.

Measurement Number	Measurement Value X_n (volts)
1	98
2	102
3	101
4	97
5	100
6	103
7	98
8	106
9	107
10	99

The average value for the set of measurements is equal to the sum of the ten measurements divided by 10. which is 101.1. The precision of the fourth measurement is

$$\text{Precision} = 1 - \left|\frac{X_n - \bar{X}_n}{\bar{X}_n}\right|$$

$$= 1 - \left|\frac{97 - 101.1}{101.1}\right| = 1 - 0.04 = 0.96$$

Example 1-3

The voltage drops across two resistors in a series circuit are measured as

$$V_1 = 6.31 \text{ V}$$

$$V_2 = 8.736 \text{ V}$$

1-To determine the applied voltage accurately, it's best to first sum the values without rounding each number individually and then round off the final result to the appropriate number of significant figures. Following this method, the applied voltage is calculated as:

E=6.31 V+8.736V = 15.046 v

Rounded to the same precision

supply voltage is calculated as: E=15.05V

2-Given that the current in the series circuit is measured at 0.0148 A, we can use this current value along with the voltage drops to calculate the resistance of each resistor.

$$R_1 = \frac{V_1}{I} = \frac{6.31\ \text{V}}{0.0148\ \text{A}} = 426\ \Omega$$

$$R_2 = \frac{V_2}{I} = \frac{8.736\ \text{V}}{0.0148\ \text{A}} = 590.3\ \Omega$$

3-The power dissipated by each resistor is calculated by multiplying the voltage drop across the resistor by the current flowing through it.

$$P_1 = V_1 I$$
$$= (6.31\ \text{V})(0.0148\ \text{A}) = 0.093\ \text{W}$$

$$P_2 = V_2 I$$
$$= (8.736\ \text{V})(0.0148\ \text{A}) = 0.1293\ \text{W}$$

1-6 ERRORS

The accuracy and precision of measurements rely not only on the quality of the measuring instrument but also on the skill of the person using it. Despite the instrument's quality and the user's care, some level of error is inevitable in measuring physical quantities. Error can be defined as the deviation of a reading or set of readings from the expected value of the measured variable. Errors are typically classified into three main categories.

1-6.1 Gross Errors

Gross errors are typically caused by the user of the instruments and result from mistakes such as misreading the instruments, incorrectly recording experimental data, or improper use of the equipment.

1-6.2 Systematic Errors

Systematic errors arise from issues with instruments, environmental conditions, or observational practices, and they tend to recur if multiple measurements of the same quantity are taken under consistent conditions.

1. **Instrument Errors:** These errors stem from problems with the instrument itself, such as friction in the meter's bearings, incorrect spring tension, improper calibration, or faulty equipment. Instrument errors can be

minimized through proper maintenance, careful use, and correct handling of the instruments.

2. **Environmental Errors:** The conditions in which instruments are used can contribute to errors. Exposure to extreme temperatures, pressures, humidity, or strong electrostatic or electromagnetic fields can adversely affect instrument performance, leading to errors.
3. **Observational Errors:** These errors occur due to the observer's actions. Common types include parallax errors from reading a meter scale from an incorrect angle and estimation errors when interpreting measurements from a scale.

1-6.3 Random Errors

Random errors are those that persist after gross and systematic errors have been significantly minimized or addressed. These errors arise from a multitude of minor, unpredictable factors and are primarily a concern in measurements requiring high precision. They can only be analyzed through statistical methods.

- **Instrument:** A device or mechanism used to determine the current value of the quantity being observed.
- **Measurement:** The process of determining the amount, quantity, degree, or capacity of a variable by comparing it directly or indirectly with accepted standards within a given system of units.
- **Expected Value:** The design value, or "most probable value," that calculations predict one should expect to measure.
- **Accuracy:** The degree to which a measurement compared to the expected value or the most probable value of the variable being measured.
- **Resolution:** The smallest detectable change in a measured variable that an instrument can register.
- **Precision:** A measure of the consistency or repeatability of measurements.

1-7 STATISTICAL ANALYSIS

When measuring a physical quantity, numerous factors can affect the results. For instance, when measuring the resistance of a wire, several factors influence the resistance value obtained. Some factors are crucial, while others may have minimal impact. Key considerations include the type and purity of the wire material, its temperature, length, cross-sectional area, current distribution, manufacturing practices (such as heat treatment), and the tension applied to the wire during measurement.

Ideally, one should understand how each parameter affects the measurement to explain and interpret the results accurately. Variations in measurements from

repeated trials can occur if the experimenter fails to control all these factors consistently.

Assuming that the effects of parameters and small variations in parameters we try to keep constant are random, we can infer that these variations have an equal chance of being positive or negative. This randomness enables us to perform a statistical analysis of our experimental data. By evaluating the mean value, average deviation, and standard deviation of the data, we can quantitatively assess the variations or errors present. This statistical approach provides a basis for judging the quality and reliability of our measurements.

An average is a value that represents a typical or most representative point within a data set. Because these typical values generally cluster around the center when the data is ordered by magnitude, averages are also referred to as measures of central tendency.

The arithmetic mean is the most commonly used type of average. For a set of n numbers $x_1, x_2, \ldots, x_n$, the arithmetic mean is defined as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Deviation refers to the difference between each individual data point and the arithmetic mean.

$$d_1 = x_1 - \bar{x}$$
$$d_2 = x_2 - \bar{x}$$
$$d_n = x_n - \bar{x}$$

The total of the deviations from the arithmetic mean for a set of numbers equals zero.

Example 1-4

For the given data, compute the following:

(a) The arithmetic mean.

(b) The deviation of each value.

(c) The algebraic sum Of the deviations.

$$x_1 = 50.1$$
$$x_2 = 49.7$$
$$x_3 = 49.6$$
$$x_4 = 50.2$$

(a) The arithmetic mean is computed as

$$\bar{x} = \frac{50.1 + 49.7 + 49.6 + 50.2}{4}$$
$$= \frac{199.6}{4} = 49.9$$

(b) The deviation of each value from the mean is computed as

$$d_1 = 50.1 - 49.9 = 0.2$$
$$d_2 = 49.7 - 49.9 = -0.2$$
$$d_3 = 49.6 - 49.9 = -0.3$$
$$d_4 = 50.2 - 49.9 = 0.3$$

(c) The algebraic sum of the deviations is

$$d_{tot} = 0.2 - 0.2 - 0.3 + 0.3 = 0$$

The extent to which numerical data spread around the average value is known as the variation or dispersion of the data. One way to measure this variation is through the average deviation. This metric can indicate the precision of a measuring instrument, with a lower average deviation reflecting higher precision. The average deviation D of a set of numbers is defined as follows:

$$D = \frac{|d_1| + |d_2| + \cdots + |d_n|}{n}$$

The average deviation is computed as follows

$$D = \frac{|0.2| + |-0.2| + |-0.3| + |0.3|}{4}$$
$$= \frac{1.0}{4} = 0.25$$

The standard deviation S of a set of values measures the extent to which the values deviate from the average value. For a set of n numbers, the standard deviation is defined as:

$$S = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{4}}$$

For small sample sizes (n < 30), the denominator is often adjusted to n−1 to provide a more accurate estimate of the standard deviation.

Example 1-5

Calculate the standard deviation for the data from Example 1-4.

$$S = \sqrt{\frac{(0.2)^2 + (0.2)^2 + (0.3)^2 + (0.3)^2}{4-1}}$$
$$= \sqrt{\frac{0.4 + 0.4 + 0.09 + 0.09}{3}}$$
$$= \sqrt{\frac{0.26}{3}} = 0.294$$

1-8 LIMITING ERRORS

Manufacturers of measuring instruments often specify accuracy as a percentage of the full-scale reading. For instance, a voltmeter might be rated as accurate within ± 2% of full-scale deflection. This specification, known as the limiting error, ensures that the measurement will be within 2% of the true value at full scale. However, for readings below full scale, the limiting error increases, so it is crucial to take measurements as close to full scale as possible.

Example 1-6

A 300-V voltmeter is rated for ±2% accuracy at full scale. Determine the limiting error when the voltmeter is used to measure a 120-V source.

The magnitude of the limiting error is

$$0.02 \times 300\ \text{V} = 6\ \text{V}$$

Therefore, the limiting error at 120 V is

$$\frac{6}{120} \times 100\% = 5\%$$

Example 1-7

A voltmeter and an ammeter are used to measure the power dissipated in a resistor, with both instruments guaranteed to be accurate within ±1% at full scale. If the voltmeter reads 80 V on its 150-V range and the ammeter reads 70 mA on its 100-mA range, calculate the limiting error for the power calculation.

The magnitude of the limiting error for the voltmeter is

$$0.01 \times 150\ \text{V} = 1.5\ \text{V}$$

The limiting error at 80 V is

$$\frac{1.5}{80} \times 100\% = 1.86\%$$

The magnitude of the limiting error for the ammeter is

$$0.01 \times 100\ \text{mA} = 1\ \text{mA}$$

The limiting error at 70 mA is

$$\frac{1}{70} \times 100\% = 1.43\%$$

The limiting error for the power calculation is the *sum* of the individual limiting errors involved; therefore,

$$\text{Limiting error} = 1.86\% + 1.43\% = 3.29\%$$

Chapter 2

Direct-Current (DC) & Alternating-current (AC) Meters

2-1 Instructional Objectives

This chapter aims to introduce the d'Arsonval meter movement and its use in ammeters, voltmeters, and ohmmeters. It also covers the characteristics of various types of meter movements used to measure alternating current (AC).

2-2 Introduction

The origin of the basic meter movement used in direct-current (DC) measurements dates back to Hans Oersted's 1820 discovery of the link between current and magnetism. Over the following fifty years, various devices utilizing this discovery were developed. In 1881, Jacques d'Arsonval patented the moving-coil galvanometer. The fundamental design created by d'Arsonval is still employed in modern meter movements. This basic moving-coil system, commonly known as the d'Arsonval meter movement or the permanent magnet moving-coil (PMMC) meter movement.

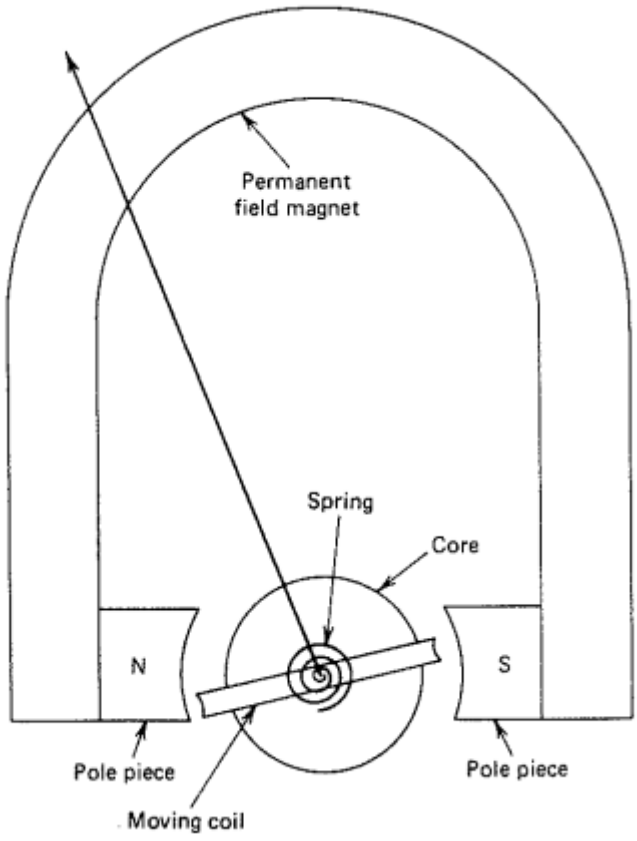


FIGURE 2-1 The d'Arsonval meter movement.

2-3 The D'Arsonval Meter Movement

The d'Arsonval meter movement remains widely used today, and this chapter provides a detailed overview of its construction and operating principle. As illustrated in Fig. 2-2, the typical commercial meter movement is based on the fundamental principles of a DC motor.

In Fig. 2-1, you can see a horseshoe-shaped permanent magnet with soft iron pole pieces. Positioned between these north-south poles is a cylindrical soft iron core, around which a coil of fine wire is wound. This coil is mounted on a very lightweight metal frame, which is set in a jewel bearing allowing it to rotate freely. A pointer attached to the moving coil indicates the measurement by deflecting up the scale as the coil rotates.

Current from the circuit under measurement flows through the windings of the moving coil.

The current flowing through the coil creates an electromagnet with its own north and south poles. These poles interact with the poles of the permanent magnet, causing the coil to rotate. Consequently, the pointer moves up the scale when current flows in the correct direction through the coil. For this reason, all DC meter movements have polarity markings.

It is important to note that the d'Arsonval meter movement is a current-responsive device. Regardless of the units (volts, ohms, etc.) for which the scale is calibrated, the moving coil's response is based on the amount of current passing through its windings.

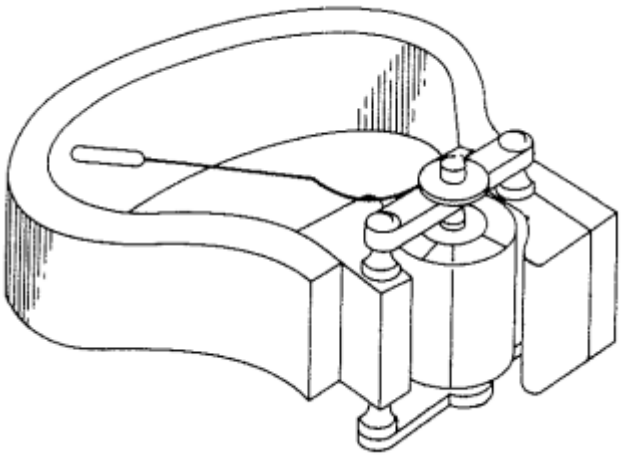


Figure 2-2 Cross-sectional view of the d'Arsonval meter

2-4 d'Arsonval Meter Movement Used in a DC Ammeter

Since the windings of the moving coil, as shown in Fig. 2-2, are made of very fine wire, the basic d'Arsonval meter movement has limited usefulness without modification. To extend the current range that the basic meter movement can measure, a low-resistance component is added in parallel with the meter's resistance, R_m. This component, called a shunt R_{sh}, provides an alternate path for the total current around the meter movement. The basic DC ammeter circuit is illustrated in Fig. 2-3. In most circuits, the current through the meter movement itself is much smaller than the current through the shunt. The resistance of the shunt is determined using Ohm's law, where:

R_{sh}= resistance of the shunt

R_m = internal resistance of the meter movement (resistance of the moving coil)

I_{sh} = current through the shunt

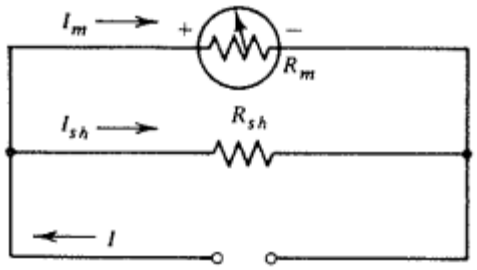


FIGURE 2-3 d'Arsonval meter movement used in an ammeter circuit.

I_m= full-scale deflection current of the meter movement

I = full-scale deflection current of the ammeter

The voltage drop across the meter movement is given by:

$V_m = I_m R_m$

Since the shunt resistor is connected in parallel with the meter movement, the voltage drop across the shunt resistor is the same as across the meter movement. Therefore:

The current through the shunt resistor is the total current minus the current through the meter movement.

The current through the shunt is calculated using the formula:

$$I_{sh} = I - I_m$$

By measuring the voltage across and the current through the shunt, we can calculate the shunt resistance.

$$R_{sh} = \frac{V_{sh}}{I_{sh}} = \frac{I_m R_m}{I_{sh}} = \frac{I_m}{I_{sh}} R_m = \frac{I_m}{I - I_m} \times R_m \ (\Omega)$$

Example 2-1

Determine the shunt resistance needed to modify a 1-mA meter movement with a 100 Ω internal resistance into a 0 to 10-mA ammeter.

$$V_m = I_m R_m = 1 \text{ mA} \times 100 \ \Omega = 0.1 \text{ V}$$

$$V_{sh} = V_m = 0.1 \text{ V}$$

$$I_{sh} = I - I_m = 10 \text{ mA} - 1 \text{ mA} = 9 \text{ mA}$$

$$R_{sh} = \frac{V_{sh}}{I_{sh}} = \frac{0.1 \text{ V}}{9 \text{ mA}} = 11.11 \ \Omega$$

The purpose of designing the shunt circuit is to enable the measurement of a current I that is n times greater than I_m. The factor n, known as the multiplying factor, links the total current to the meter current.

$$I = nI_m$$

$$R_{sh} = \frac{R_m I_m}{nI_m - I_m}$$

$$= \frac{R_m}{n - 1} \ (\Omega)$$

Example 2-2

A 100-µA meter movement with an internal resistance of 800 Ω is used to create a 0- to 100-mA ammeter. Calculate the required shunt resistance.

The multiplying factor n is the ratio of 100 mA to 100 µA.

$$n = \frac{I}{I_m} = \frac{100 \text{ mA}}{100 \ \mu\text{A}} = 1000$$

$$R_{sh} = \frac{R_m}{n-1} = \frac{800\,\Omega}{1000-1} = \frac{800}{999} \approx 0.80\,\Omega$$

2-5 THE AYRTON SHUNT

The shunt resistance is effective for single-range ammeters. However, for multi-range ammeters, the Ayrton shunt, or universal shunt, is often a more appropriate choice. One key advantage of the Ayrton shunt is that it prevents the meter movement from being part of the circuit without any shunt resistance. Additionally, it can be used with a broad range of meter movements. The Ayrton shunt circuit is depicted in Fig. 2-4.

To calculate the individual resistance values for the shunts, you start with the most sensitive range and proceed to the least sensitive range. In Fig. 2-4, the most sensitive range is the 1-A range. For this range, the shunt resistance is given by $R_{sh} = R_a + R_b + R_c$. The shunt resistance for this range can be computed using this Equation.

$$R_{sh} = \frac{R_m}{n-1}$$

The equations to calculate the values of each shunt, R_a, R_b, and R_c, can be derived by examining Fig. 2-5. Since the resistance R_b+R_c is in parallel with $R_m + R_a$, the voltage across each parallel branch must be the same, and this can be expressed as:

$$V_{R_b+R_c} = V_{R_a+R_m}$$

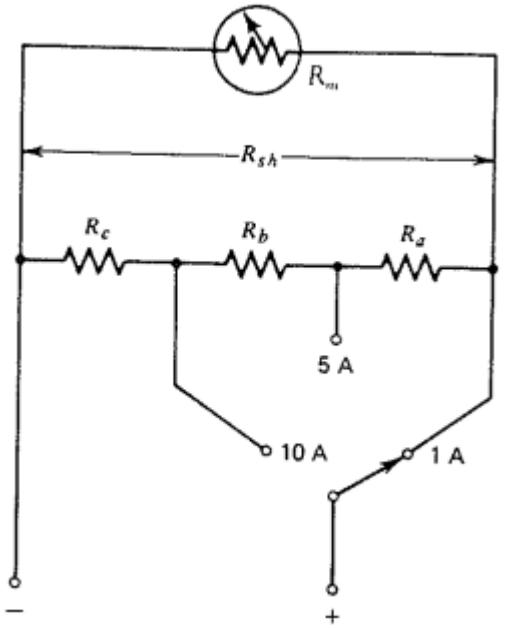


Figure 2-4 an ammeter using Ayrton shunt

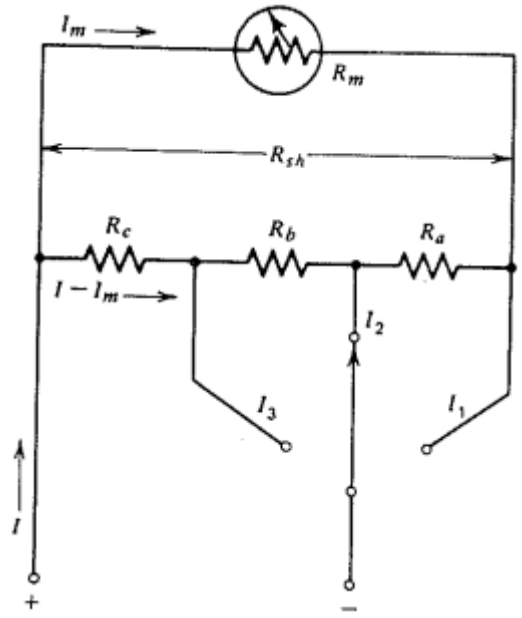


Figure 2-5 The Ayrton shunt circuit

$$(R_b + R_c)(I_2 - I_m) = I_m(R_a + R_m)$$

$$I_2(R_b + R_c) - I_m(R_b + R_c) = I_m[R_{sh} - (R_b + R_c) + R_m]$$

$$I_2(R_b + R_c) - I_m(R_b + R_c) = I_m R_{sh} - I_m(R_b + R_c) + I_m R_m$$

$$R_b + R_c = \frac{I_m(R_{sh} + R_m)}{I_2} \ (\Omega)$$

$$R_a = R_{sh} - (R_b + R_c) \ (\Omega)$$

$$R_c = \frac{I_m(R_{sh} + R_m)}{I_3}$$

$$R_b = (R_b + R_c) - R_c \ (\Omega)$$

Example 2-3

Determine values of shunt resistors for figure 2-6.

$$R_{sh} = \frac{R_m}{n-1} = \frac{1 \text{ k}\Omega}{100-1} = \frac{1 \text{ k}\Omega}{99} = 10.1 \ \Omega$$

$$R_b + R_c = \frac{I_m(R_{sh} + R_m)}{I_2}$$

$$= \frac{(100 \ \mu\text{A})(10.1 \ \Omega + 1 \text{ k}\Omega)}{100 \text{ mA}} = 1.01 \ \Omega$$

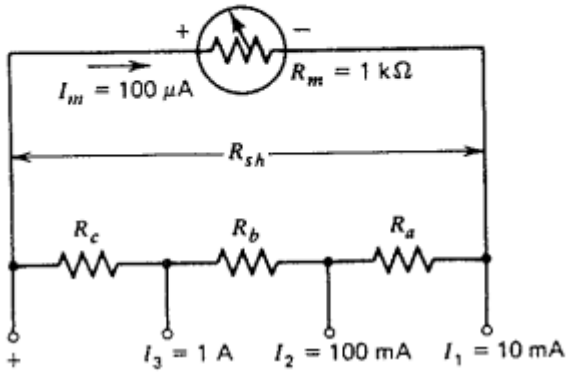


Figure 2-6 The Ayrton shunt circuit

$$R_c = \frac{I_m(R_{sh} + R_m)}{I_3}$$

$$= \frac{(100 \ \mu\text{A})(10.1 \ \Omega + 1 \text{ k}\Omega)}{1 \text{ A}} = 0.101 \ \Omega$$

$$R_b = (R_b + R_c) - R_c = 1.01 \ \Omega - R_c = 1.01 \ \Omega - 0.101 \ \Omega = 0.909 \ \Omega$$

$$R_a = R_{sh} - (R_b + R_c)$$
$$= 10.1\ \Omega - (0.909\ \Omega + 0.101\ \Omega) = 9.09\ \Omega$$

$$R_{sh} = R_a + R_b + R_c = 9.09\ \Omega - 0.909\ \Omega + 0.101\ \Omega = 10.1\ \Omega$$

2-6 D'Arsonval Meter Movement Used in a DC Voltmeter

The basic d'Arsonval meter movement can be transformed into a DC voltmeter by connecting a series resistor, known as a multiplier R_s, to the meter movement, as illustrated in Fig. 2-7. The multiplier's role is to increase the voltage range of the meter and to ensure that the current through the d'Arsonval meter movement does not exceed the maximum current required for full-scale deflection. To determine the appropriate value for the multiplier resistor, you must first calculate the sensitivity of the meter movement. Sensitivity is defined as the reciprocal of the full-scale deflection current.

$$\text{Sensitivity} = \frac{1}{I_{fs}}\ (\Omega/\text{V})$$

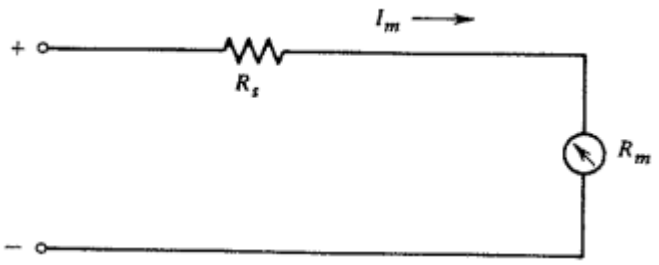


Figure 2-7 D'Arsonval meter movement used in a dc voltmeter

When measuring voltage, the voltmeter is connected across the resistance being measured. This setup places the entire resistance of the voltmeter in parallel with the circuit resistance. Therefore, it's important for the voltmeter's resistance to be significantly higher than that of the circuit. Due to the use of different meter movements in various voltmeters and the varying multiplier values for each range, expressing the total resistance of the instrument as a single rating can be challenging. A more practical way to convey this information is through the instrument's sensitivity rating, usually printed on the meter's face. This rating indicates the resistance of the voltmeter for a one-volt range. To find the total resistance that the voltmeter introduces to a circuit, simply multiply the sensitivity by the range.

Example 2-4

Calculate the sensitivity of a 100-µA meter movement intended for use as a DC voltmeter.

The sensitivity is calculated as follows:

$$S = \frac{1}{I_{fs}} = \frac{1}{100\ \mu A} = 10\ \frac{k\Omega}{V}$$

The units of sensitivity express the value of the multiplier resistance for the 1 -V range. To calculate the value of the multiplier for voltage ranges greater than 1 V. simply multiply the sensitivity by the range and subtract the internal resistance of the meter movement, or

$$R_s = S \times \text{Range} - \text{Internal Resistance}$$

EXAMPLE 2-5

Calculate the value of the multiplier resistance for the 50V range of a DC voltmeter that uses a 500-µA meter movement with an internal resistance of 1 kohm.

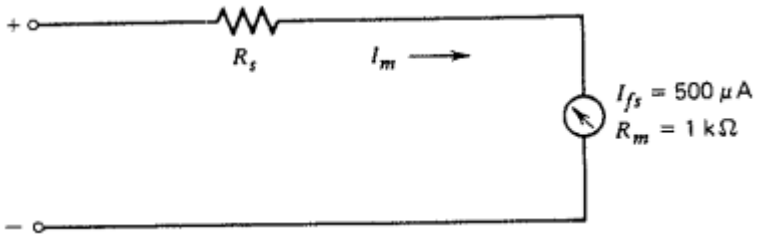


Figure 2-8 DC voltmeter

$$S = \frac{1}{I_{fs}} = \frac{1}{500\ \mu A} = 2\ \frac{k\Omega}{V}$$

$$R_s = S \times \text{Range} - R_m$$
$$= \frac{2\ k\Omega}{V} \times 50\ V - 1\ k\Omega = 100\ k\Omega - 1\ k\Omega = 99\ k\Omega$$

Example 2-6

Determine the value of the multiplier resistances for the multi-range DC voltmeter circuit depicted in Fig. 2-9.

$$S = \frac{1}{I_{fs}} = \frac{1}{50\ \mu A} = 20\ \frac{k\Omega}{V}$$

On the 3-V range,

$$R_{s1} = S \times \text{Range} - R_m$$
$$= \frac{20\ k\Omega}{V} \times 3\ V - 1\ k\Omega = 59\ k\Omega$$

On the 10-V range,

$$R_{s2} = S \times \text{Range} - R_m$$
$$= \frac{20\ k\Omega}{V} \times 10\ V - 1\ k\Omega = 199\ k\Omega$$

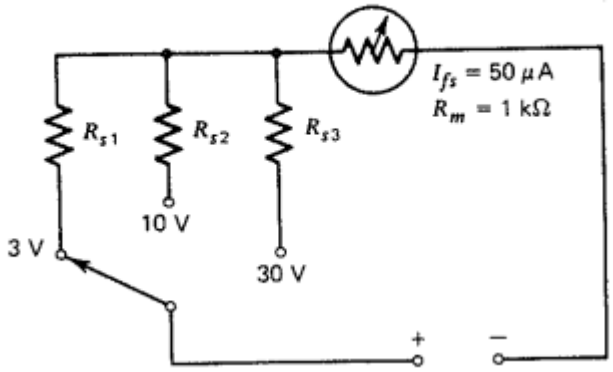


Figure 2-9 multi-range DC voltmeter circuit

On the 30-V range,

$$R_{s3} = S \times \text{Range} - R_m$$
$$= \frac{20\ k\Omega}{V} \times 30\ V - 1\ k\Omega = 599\ k\Omega$$

Example 2-7

Calculate the value of the multiplier resistors for the multiple-range dc voltmeter circuit shown in Fig. 2-10.

The sensitivity of the meter movement is computed as

$$S = \frac{1}{I_{fs}} = \frac{1}{50\ \mu A} = 20\ k\Omega/V$$

(a) On the 3-V range,

$$R_a = S \times \text{Range} - R_m = \frac{20\ \text{k}\Omega}{\text{V}} \times 3\ \text{V} - 1\ \text{k}\Omega$$

$$= 59\ \text{k}\Omega$$

(b) On the 10-V range,

$$R_b = S \times \text{Range} - (R_a + r_m) = \frac{20\ \text{k}\Omega}{\text{V}} \times 10\ \text{V} - 60\ \text{k}\Omega$$

$$= 140\ \text{k}\Omega$$

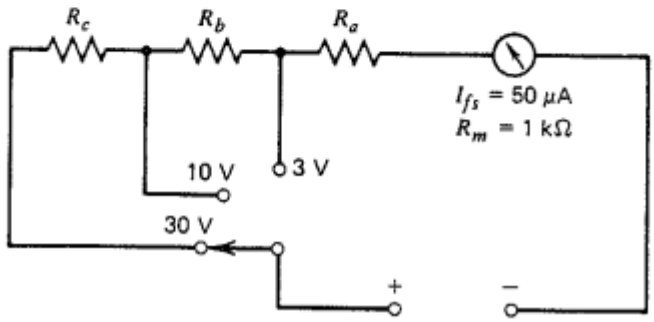


Figure 2-10 multiple-range dc voltmeter circuit

(c) On the 30-V range,

$$R_c = S \times \text{Range} - (R_a + R_b + R_m) = \frac{20\ \text{k}\Omega}{\text{V}} \times 30\ \text{V} - 200\ \text{k}\Omega$$

$$= 400\ \text{k}\Omega$$

2-7 Voltmeter Loading Effect

When a voltmeter is used to measure the voltage across a circuit component, the voltmeter itself is connected in parallel with the component. Since the combined resistance of two parallel resistors is lower than the resistance of either resistor alone, the overall resistance seen by the source decreases when the voltmeter is connected. Consequently, the voltage across the component drops whenever the voltmeter is in place. This reduction in voltage may be insignificant or considerable, depending on the sensitivity of the voltmeter being used. This phenomenon is known as voltmeter loading, and it is demonstrated in the following examples. The associated error is referred to as loading error.

Example 2-8

Two different voltmeters are used to measure the voltage across resistor R_B in the circuit shown in Fig. 2-11. The meters are as follows:

Meter A: $S = 1\ \text{k}\Omega/\text{V}$, $R_m = 0.2\ \text{k}\Omega$, range = 10 V

Meter B: $S = 20\ \text{k}\Omega/\text{V}$, $R_m = 1.5\ \text{k}\Omega$, range = 10 V

Calculate

(a) Voltage across R_B without any meter connected across it.

(b) Voltage across R_B when meter A is used.

(c) Voltage across R_B when meter B is used.

(d) Error in voltmeter readings.

(a) The voltage across resistor R_B without either meter connected is found using the voltage divider equation:

$$V_{R_B} = E\frac{R_B}{R_A + R_B}$$

$$= 30\ \text{V} \times \frac{5\ \text{k}\Omega}{25\ \text{k}\Omega + 5\ \text{k}\Omega} = 5\ \text{V}$$

(b) Starting with meter A, the total resistance it presents to the circuit is

$$R_{T_A} = S \times \text{Range}$$

$$= \frac{1\ \text{k}\Omega}{\text{V}} \times 10\ \text{V} = 10\ \text{k}\Omega$$

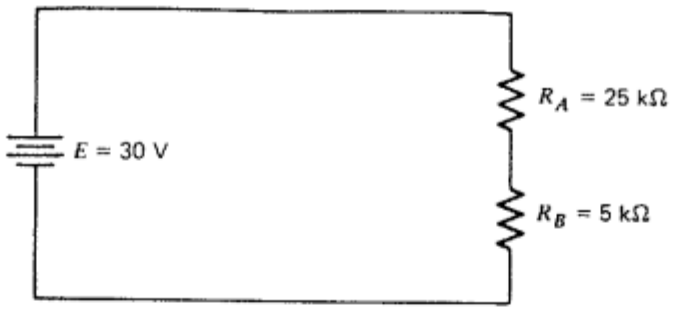


FIGURE 2-11 Circuit for Example 2-8 showing voltmeter loading.

The parallel combination of R_B and meter A is

$$R_{e1} = \frac{R_B \times R_{T_A}}{R_B + R_{T_A}}$$

$$= \frac{5\ \text{k}\Omega \times 10\ \text{k}\Omega}{5\ \text{k}\Omega + 10\ \text{k}\Omega} = 3.33\ \text{k}\Omega$$

Therefore, the voltage reading obtained with meter A, determined by the voltage divider equation, is

$$V_{R_B} = E\frac{R_{e1}}{R_{e1} + R_A}$$

$$= 30\ \text{V} \times \frac{3.33\ \text{k}\Omega}{3.33\ \text{k}\Omega + 25\ \text{k}\Omega}\ 3.53\ \text{V}$$

c) The total resistance that meter B presents to the circuit is

$$R_{T_B} = S \times \text{Range}$$

$$= \frac{20\ \text{k}\Omega}{\text{V}} \times 10\ \text{V} = 200\ \text{k}\Omega$$

The parallel combination of R_B and meter B is

$$R_{e2} = \frac{R_B \times R_{T_B}}{R_B + R_{T_B}}$$

$$= \frac{5\ \text{k}\Omega \times 200\ \text{k}\Omega}{5\ \text{k}\Omega + 200\ \text{k}\Omega} = 4.88\ \text{k}\Omega$$

Therefore, the voltage reading obtained with meter B, determined by

use of the voltage divider equation, is

$$V_{R_B} = E\frac{R_{e2}}{R_{e2} + R_A}$$

$$= 30\text{ V} \times \frac{4.88\text{ k}\Omega}{4.88\text{ k}\Omega + 25\text{ k}\Omega} = 4.9\text{ V}$$

(d) $$\text{Voltmeter } A \text{ error} = \frac{5\text{ V} - 3.53\text{ V}}{5\text{ V}} \times 100\% = 29.4\%$$

$$\text{Voltmeter } B \text{ error} = \frac{5\text{ V} - 4.9\text{ V}}{5\text{ V}} \times 100\% = 2\%$$

2-8 The Ohmmeter

The basic d'Arsonval meter movement can also be used with a battery and a resistor to create a simple ohmmeter circuit, as shown in Fig. 2-12. When points X and Y are connected, a simple series circuit is formed, with the current flowing through the meter movement due to the voltage source E. The current's amplitude is limited by resistors R_z and R_m. In Fig. 2-12, notice that resistor R_z consists of both a fixed and a variable portion.

Connecting points X and Y is equivalent to shorting the test probes of an ohmmeter together to "zero" the instrument before use. This is a normal procedure when operating an ohmmeter. After connecting points X and Y, the variable part of resistor R_z is adjusted to achieve full-scale deflection on the meter movement.

The current through the meter movement can be calculated using Ohm's law as follows:

$$I_{fs} = \frac{E}{R_z + R_m}$$

To determine the value of the unknown resistor, R_x, we connect it between points X and Y in Fig. 2-12. The circuit current is now given by:

$$I = \frac{E}{R_z + R_m + R_x}$$

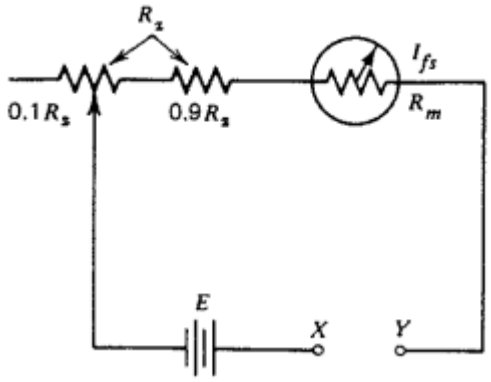


Figure 2-12 a simple ohmmeter circuit

The current I is less than the full-scale deflection current due to the additional resistance R_x. The ratio of the current I to the full-scale deflection current is equal to the ratio of the circuit resistances and can be expressed as:

$$\frac{I}{I_{fs}} = \frac{E/(R_z + R_m + R_x)}{E/(R_z + R_m)} = \frac{R_z + R_m}{R_z + R_m + R_x}$$

If we denote P as the ratio of the current to the full-scale deflection current, then we can state that:

$$P = \frac{I}{I_{fs}} = \frac{R_z + R_m}{R_z + R_m + R_x}$$

Example 2-9

A meter movement with a full-scale deflection current of 1 mA is to be used in an ohmmeter circuit. This meter movement has an internal resistance R_m of 100 Ω, and a 3-V battery will be used in the circuit. Mark off the meter for resistance readings.

Table 2-1 scale on ohmmeter

P (%)	R_x (kΩ)	$R_z + R_m$ (kΩ)
20	12	3
40	4.5	3
50	3	3
75	1	3
100	0	3

$$R_z = \frac{E}{I_{fs}} - R_m$$

$$R_z = \frac{3\text{ V}}{1\text{ mA}} - 100\ \Omega = 2.9\ \text{k}\Omega$$

The value of R_x with 20% full-scale deflection is

$$R_x = \frac{R_z + R_m}{P} - (R_z + R_m)$$

$$= \frac{2.9\ \text{k}\Omega + 0.1\ \text{k}\Omega}{0.2} - (2.9\ \text{k}\Omega + 0.1\ \text{k}\Omega)$$

$$= \frac{3\ \text{k}\Omega}{0.2} - 3\ \text{k}\Omega = 12\ \text{k}\Omega$$

The value of R_x with 40% full-scale deflection is

$$R_x = \frac{R_z + R_m}{P} - (R_z + R_m)$$

$$= \frac{3\ \text{k}\Omega}{0.4} - 3\ \text{k}\Omega = 4.5\ \text{k}\Omega$$

The value of R_x with 50% full-scale deflection is

$$R_x = \frac{R_z + R_m}{P} - (R_z + R_m)$$

$$= \frac{3\ \text{k}\Omega}{0.5} - 3\ \text{k}\Omega = 3\ \text{k}\Omega$$

The value of R_x with 75% full-scale deflection is

$$R_x = \frac{R_z + R_m}{P} - (R_z + R_m)$$

$$= \frac{3\ \text{k}\Omega}{0.75} - 3\ \text{k}\Omega = 1\ \text{k}\Omega$$

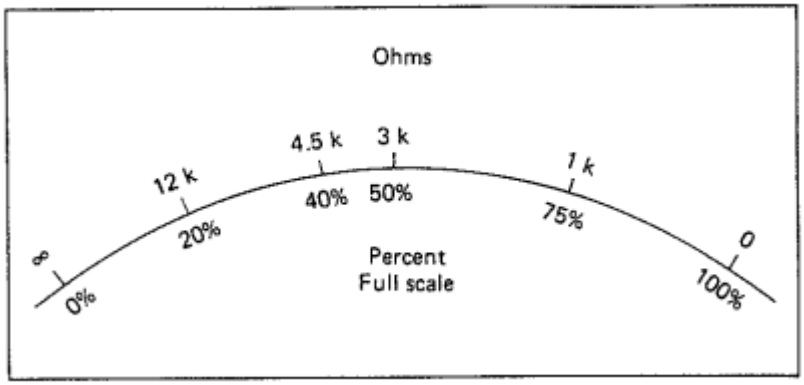


Figure 2-13 ohmmeter scale

Example 2-10

An ohmmeter utilizes a 1.5-V battery and a basic 50-μA movement. Calculate:

(a) The value of R_z required.

(b) The value of R_x that would produce half-scale deflection in the circuit.

(a) The appropriate value of R_z is calculated as follows:

$$R_z = \frac{E}{I_{fs}} - R_m$$

$$= \frac{1.5\ \text{V}}{50\ \mu\text{A}} - 1\ \text{k}\Omega = 29\ \text{k}\Omega$$

(b)

$$R_x = R_z + R_m = 30\ \text{k}\Omega$$

2-9 Multiple-Range Ohmmeters

The ohmmeter circuit described previously is not suitable for measuring resistance over a wide range of values. To address this, multiple-range ohmmeters are used. One approach to constructing a multiple-range ohmmeter is shown in Fig. 2-20. This instrument employs a basic 50-μA meter movement with an internal resistance of 2 kΩ. An additional resistance of 28 kΩ is provided by R_z, which includes both a fixed resistor and a zeroing potentiometer. R_z is essential to limit the current through the meter movement to 50 μA when the test probes (not shown) connected to X and Y are shorted together.

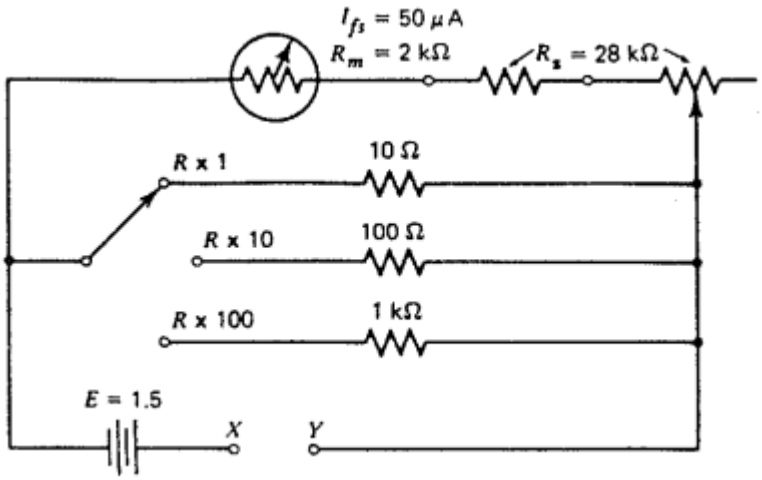


Figure 2-14 a multiple-range ohmmeter

In the R x 1 range, a 10-Ω resistor is connected in parallel with the meter movement. Therefore, the internal resistance of the ohmmeter on the R x 1 range is the parallel combination of 10 Ω and 30 kΩ, which is approximately 10 Ω. As a result, the pointer will deflect to midscale when a 10-Ω resistor is connected across X and Y.

When the instrument is set to the R x 10 range, the total resistance of the ohmmeter is 100 Ω in parallel with 30 kΩ, which is approximately 100 Ω. Consequently, the pointer deflects to midscale when a 100-Ω resistor is connected between the test probes. Midscale is marked as 10 Ω, so the resistor value is determined by multiplying the reading by the range multiplier of 10, resulting in a midscale value of 100 Ω (R x 10).

When the ohmmeter is set to the R x 100 range, the total resistance of the instrument is 1 kΩ in parallel with 30 kΩ, which remains approximately 1 kΩ. Therefore, the pointer deflects to midscale when a 1-kΩ resistor is connected across the test probes. This yields a midscale reading of 10 multiplied by 100, or 1 kΩ for the resistor.

Example 2-11

(a) In Fig. 2-15, determine the current through the meter, I_m, when a 20-Ω resistor is measured on the R x 1 range.

(b) Demonstrate that this same current flows through the meter movement when a 200-Ω resistor is measured on the R x 10 range.

(c) Show that the same current also flows when a 2 kΩ resistor is measured on the R x 100 range.

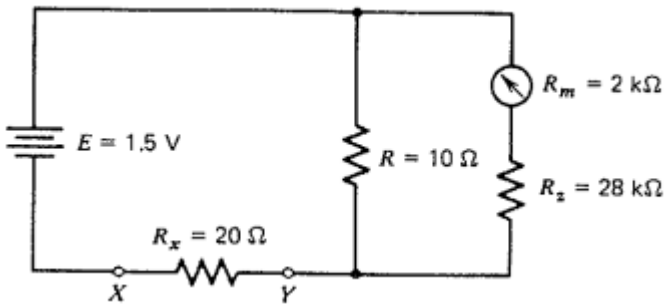


Figure 2-15 ohmmeter circuit

(a) When the ohmmeter is set to the R x 1 range, the circuit is as depicted in Fig. 2-15. The voltage across the potential divider formed by the resistances is calculated as follows:

$$V = 1.5\ \text{V} \times \frac{10\ \Omega}{10\ \Omega + 20\ \Omega} = 0.5\ \text{V}$$

The current through the meter movement is calculated as follows:

$$I_m = \frac{0.5\ \text{V}}{30\ \text{k}\Omega} = 16.6\ \mu\text{A}$$

(b) When the ohmmeter is set to the R x 10 range, the circuit is as illustrated in Fig. 2-16. The voltage across the parallel combination of resistances is calculated as follows:

$$V = 1.5\ \text{V} \times \frac{100\ \Omega}{100\ \Omega + 200\ \Omega} = 0.5\ \text{V}$$

The current through the meter movement is computed as

$$I_m = \frac{0.5\ \text{V}}{30\ \text{k}\Omega} = 16.6\ \mu\text{A}$$

(c) When the ohmmeter is set to the Rx 100 range, the circuit is as shown in Fig. 2-17. The voltage across the parallel combination of resistances is calculated as follows:

$$V = 1.5 \times \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 2\ \text{k}\Omega} = 0.5\ \text{V}$$

$$I_m = \frac{0.5\ \text{V}}{30\ \text{k}\Omega} = 16.6\ \mu\text{A}$$

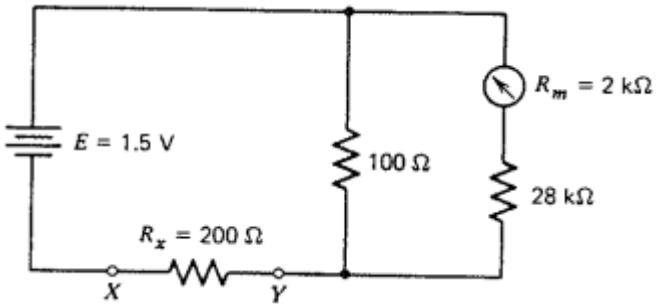


Figure 2-16 ohmmeter circuit

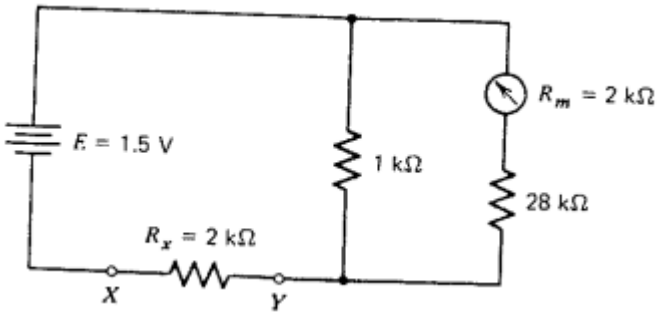


Figure 2-17 ohmmeter circuit

2-10 THE MULTIMETER

In Chapter 2, we've covered the ammeter, voltmeter, and ohmmeter. All of these instruments share a common feature: they use the same basic current-sensitive d'Arsonval meter movement. This similarity suggests that, with the right switching mechanism, the three circuits could be combined into a single instrument. The multimeter, or VOM (Volt-Ohm-Milliammeter), is precisely such a device. It is a versatile testing instrument with the necessary circuitry to measure AC or DC voltage, direct current, and resistance. A typical commercial VOM of laboratory quality is usually built around a basic 50-μA meter movement.

The Simpson Model 260, illustrated in Fig. 2-18, is a general-purpose VOM. This instrument uses a 50-μA meter movement, giving it a sensitivity of 20 kΩ/V on the DC voltage ranges. It is capable of measuring a wide range of electrical quantities, as detailed in Table 2-2.

Table 2-2

Simpson 260 Measurement Ranges

Direct current	0–50 μA, 0–1/10/100/500 mA, 0–10 A
Dc volts	0–250 mV, 0–2.6/10/50/250/1000/5000 V
Ac volts	0–2.5/100/50/250/1000/5000 V
Ohms	$R \times 1$, $R \times 100$, $R \times 10{,}000$



Figure 2-18 The Simpson Model 260

which lists 22 ranges for measuring voltage, current, and resistance as well as additional ranges for measuring audio-frequency output voltage and sound level.

Alternating-current (AC) meters

2-11 D'Arsonval Meter Movement With Half-Wave Rectification

To measure AC current with the d'Arsonval meter movement, the AC current must first be rectified using a diode rectifier to create a unidirectional current

flow. Various types of rectifiers can be used, such as a copper oxide rectifier, a vacuum diode, or a semiconductor (or "crystal") diode.

By adding a diode to the DC voltmeter circuit, as illustrated in Fig. 2-19, we can modify the circuit to measure AC voltage.

The sensitivity of a DC voltmeter is

$$S = \frac{1}{I_{fs}} = \frac{1}{1\ \text{mA}} = 1\ \text{k}\Omega/\text{V}$$

A multiplier of ten times this value means that a 10-V DC input will result in full-scale deflection when connected with the indicated polarity.

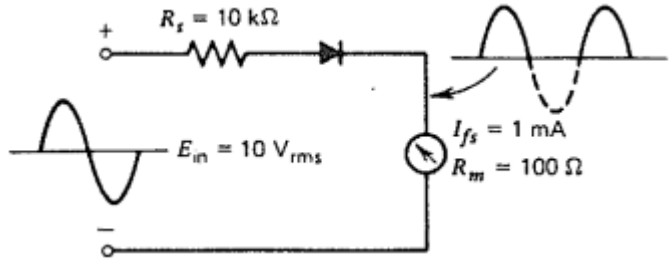


Figure 2-19 a DC voltmeter circuit that has been modified to measure AC voltage.

In Fig. 2-19, assuming an ideal diode, the forward-biased diode will not affect the operation of the circuit.

Now, if we replace the 10-V DC input with a 10-V RMS (root-mean-square) sine-wave input, the voltage across the meter movement will only reflect the positive half-cycle of the sine wave due to the diode's rectifying action. The peak value of the 10-V rms sine wave is

$$E_p = 10\ \text{V}_{\text{rms}} \times 1.414 = 14.14\ \text{V}_{\text{peak}}$$

The DC meter movement will respond to the average value of the AC sine wave, where the average, or DC value, equals 0.318 times the peak value.

$$E_{\text{ave}} = E_{\text{dc}} = 0.318 \times E_p$$

Or

$$E_{\text{ave}} = \frac{E_p}{\pi} = 0.45 \times E_{\text{rms}}$$

The diode action produces an approximate half-sine wave across the load resistor, and the average value of this voltage is referred to as the DC voltage. This is the

voltage that a DC voltmeter connected across the load resistor would read. For example, if the output voltage from a half-wave rectifier is 10 V, a DC voltmeter would indicate approximately 4.5 V. Thus, a meter that shows full-scale deflection with a 10-V DC signal will only deflect to 4.5 V when a 10-V RMS sinusoidal AC waveform is applied. This demonstrates that the AC voltmeter is less sensitive than the DC voltmeter. In fact, an AC voltmeter using half-wave rectification is only about 45% as sensitive as a DC voltmeter.

Typically, the circuit would be designed so that full-scale deflection occurs with a 10-V RMS AC input, meaning the multiplier resistor would be about 45% of the value used for a 10-V DC voltmeter. Since the equivalent DC voltage is 45% of the RMS value of the AC voltage, this relationship can be expressed with an equation for calculating the value of the multiplier resistor.

$$R_s = \frac{E_{dc}}{I_{dc}} - R_m = \frac{0.45E_{rms}}{I_{dc}} - R_m$$

We can infer that for a half-wave rectifier,

$$S_{ac} = 0.45S_{dc}$$

Example 2-12

Compute the value of the multiplier resistor for a 10-V RMS AC range on the voltmeter shown in Fig. 2-20:

We can approach the problem using these different equations and methods.

(a)

$$S_{dc} = \frac{1}{I_{fs}} = \frac{1}{1\text{ mA}} = \frac{1\text{ k}\Omega}{\text{V}}$$

$$\begin{aligned} R_s &= S_{dc} \times \text{Range}_{dc} - R_m \\ &= \frac{1\text{ k}\Omega}{\text{V}} \times \frac{0.45E_{rms}}{1} - R_m \\ &= \frac{1\text{ k}\Omega}{\text{V}} \times \frac{4.5\text{ V}}{1} - 300\ \Omega = 4.2\text{ k}\Omega \end{aligned}$$

(b)

$$S_{ac} = 0.45 S_{dc} = 0.45 \times \frac{1}{I_{fs}} = \frac{450\ \Omega}{\text{V}}$$

$$R_s = S_{ac} \times \text{Range}_{ac} - R_m$$
$$= \frac{450\ \Omega}{\text{V}} \times \frac{10\ \text{V}}{1} - 300\ \Omega = 4.2\ \text{k}\Omega$$

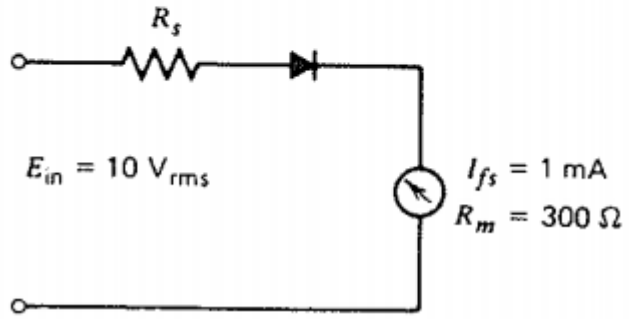


Figure 2-20 an AC voltmeter utilizing half-wave rectification.

(c)

$$R_s = \frac{0.45 E_{rms}}{I_{fs}} - R_m$$
$$= \frac{0.45 \times 10\ V_{rms}}{1\ \text{mA}} - 300\ \Omega$$
$$= \frac{4.5\ \text{V}}{1\ \text{mA}} - 300\ \Omega = 4.2\ \text{k}\Omega$$

Commercial AC voltmeters that employ half-wave rectification also include an additional diode and a shunt, as depicted in Fig. 2-21. This setup, often referred to as an instrument rectifier, features a double-diode arrangement within a single package. Diode D2 is reverse-biased during the positive half-cycle, having no effect on the circuit's operation. During the negative half-cycle, D2 becomes forward-biased, providing an alternate path for reverse-biased leakage current that would pass through the meter movement and diode D1.

The shunt resistor R_{sh} is used to increase the current flow through D1 during the positive half-cycle, allowing the diode to operate in a more linear portion of its characteristic curve. Although this shunt resistor improves the meter's linearity at lower AC voltage ranges, it also further reduces the AC sensitivity of the voltmeter.

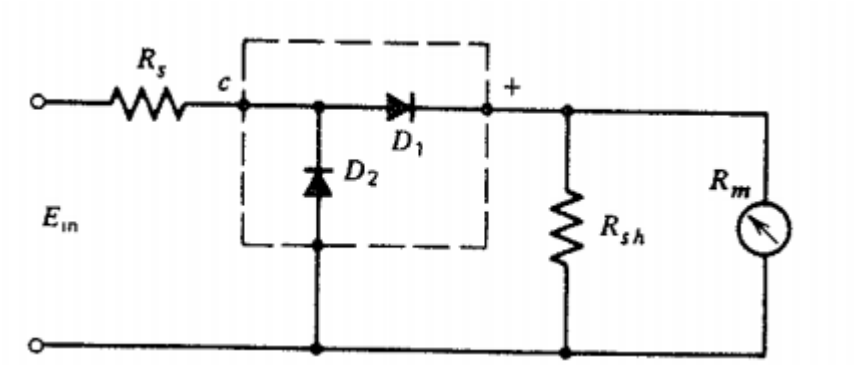


Fig. 2-21 half-wave rectification using an instrument rectifier and shunt resistor to improve linearity.

Example 2-13

In the half-wave rectifier depicted in Fig. 2-22, where diodes D1 and D2 have an average forward resistance of 50 Ω and are considered to have infinite resistance in the reverse direction, perform the following calculations:

(a) Determine the value of the multiplier resistor R_s.

(b) Calculate the AC sensitivity.

(c) Compute the equivalent DC sensitivity.

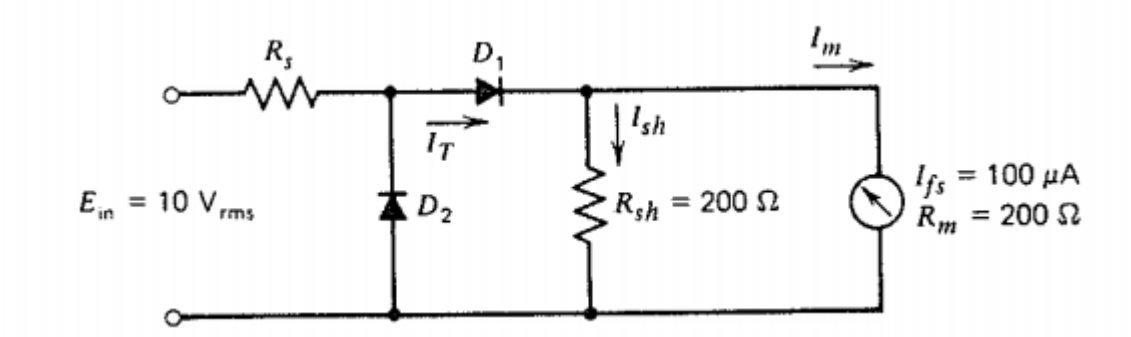


Fig. 2-22 half-wave rectification with shunt resistor

$$I_{sh} = \frac{E_m}{R_{sh}} = \frac{100\ \mu A \times 200\ \Omega}{200\ \Omega} = 100\ \mu A$$

$$I_T = I_{sh} + I_m = 100\ \mu A + 100\ \mu A = 200\ \mu A$$

$$E_{dc} = 0.45 \times E_{rms} = 0.45 \times 10\ V = 4.5\ V$$

The total resistance of the meter circuit is

$$R_T = \frac{E_{dc}}{I_T} = \frac{4.5\ V}{200\ \mu A} = 22.5\ k\Omega$$

The total resistance is made up of several separate resistances and is computed as

$$R_T = R_s + R_d + \frac{R_m R_{sh}}{R_m + R_{sh}}$$

Therefore, we can solve for R_s as

$$R_s = R_T - R_d - \frac{R_m R_{sh}}{R_m + R_{sh}}$$

$$= 22{,}500\ \Omega - 50\ \Omega - \frac{200\ \Omega \times 200\ \Omega}{200\ \Omega + 200\ \Omega}$$

$$= 22.35\ k\Omega$$

(b) The ac sensitivity is computed as

$$S_{ac} = \frac{R_T}{\text{Range}} = \frac{22{,}500\ \Omega}{10\ V} = 2250\ \Omega/V$$

(c) The dc sensitivity is computed as

$$S_{dc} = \frac{1}{I_T} = \frac{1}{200\ \mu A} = 5000\ \mu/V$$

2-12 D'Arsonval Meter Movement With Full-Wave Rectification

In many cases, a full-wave rectifier is preferred over a half-wave rectifier in AC voltmeters due to its higher sensitivity. The most commonly used circuit for full-wave rectification is the bridge-type rectifier, as shown in Fig. 2-23.

During the positive half-cycle, current flows through diode D2, through the meter movement from positive to negative, and through diode D3. The transformer secondary polarities shown in the circles indicate the positive half-cycle. Since

current flows through the meter movement during both half-cycles, we can expect the pointer deflection to be greater compared to a half-wave rectifier, which only allows current flow during every other half-cycle. As a result, the instrument using full-wave rectification will have greater sensitivity if the deflection remains the same.

Consider the circuit depicted in Fig. 2-24. The peak value of the 10-V rms signal is computed as with half wave rectifier

$$E_p = 1.414 \times E_{rms} = 14.14\ V_{peak}$$

The average, or DC value of the pulsating sine wave is

$$E_{ave} = 0.636 E_p = 9\ V$$

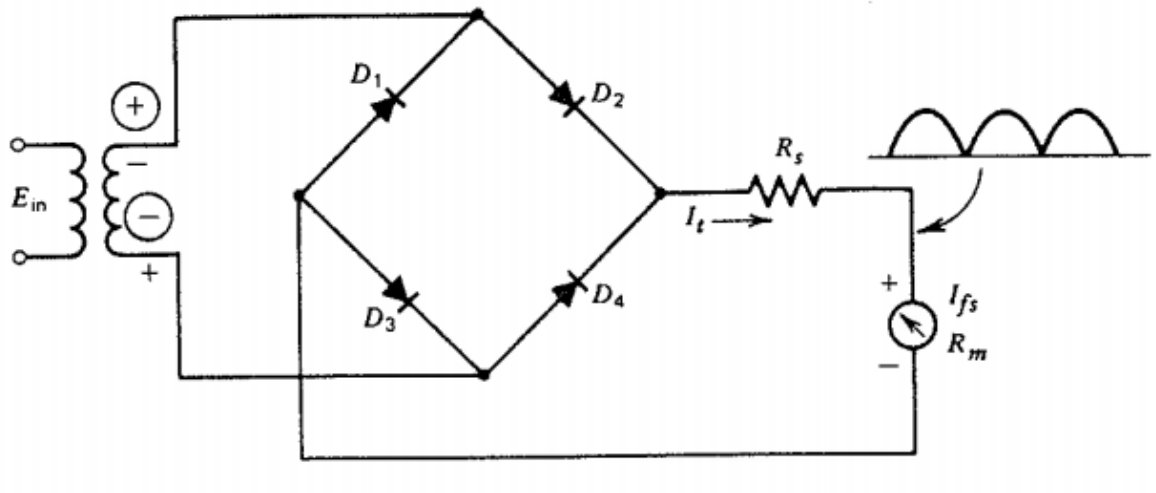


Figure 2-23 full-wave bridge rectifier used in AC voltmeter circuit

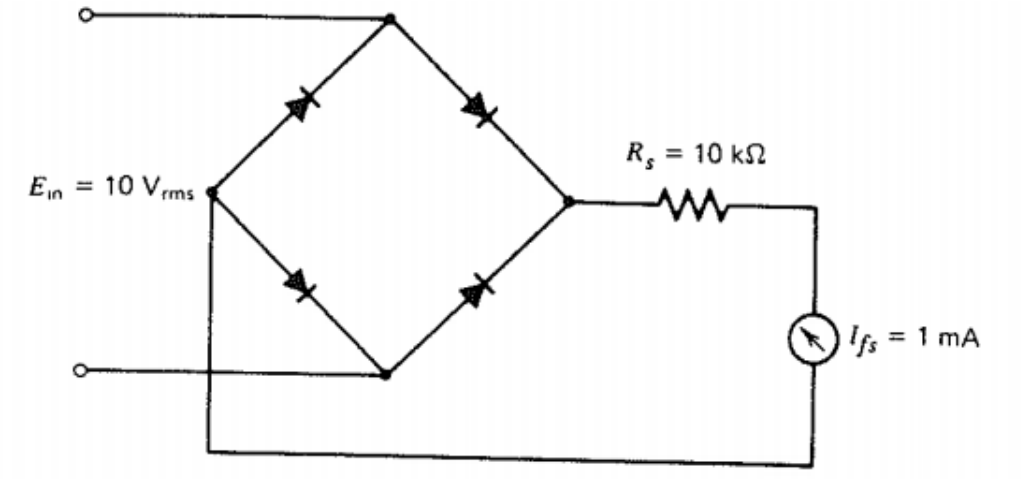


Figure 2-24 an AC voltmeter circuit employing full-wave rectification. Alternatively, the value can be computed as:

$$E_{ave} = 0.9 \times E_{rms} = 0.9 \times 10\ V = 9\ V$$

Thus, the 10-V rms voltage is equivalent to 9 V_{dc}. With full-wave rectification, the pointer will deflect to 90% of full scale. This indicates that an AC voltmeter using full-wave rectification has a sensitivity that is 90% of the DC sensitivity, or twice the sensitivity of a circuit using half-wave rectification. Similar to the half-wave rectifier, the circuit is designed for full-scale deflection, meaning the multiplier resistor would be only 90% of the value used for a 10-V DC voltmeter.

$$S_{ac} = 0.9 S_{dc}$$

Example 2-14

Calculate the value of the multiplier resistor for a 10-V rms AC range on the voltmeter shown in Fig. 2-25.

The dc sensitivity is

$$S_{dc} = \frac{1}{I_{fs}} = \frac{1}{1\ mA} = \frac{1\ k\Omega}{V} \qquad (2$$

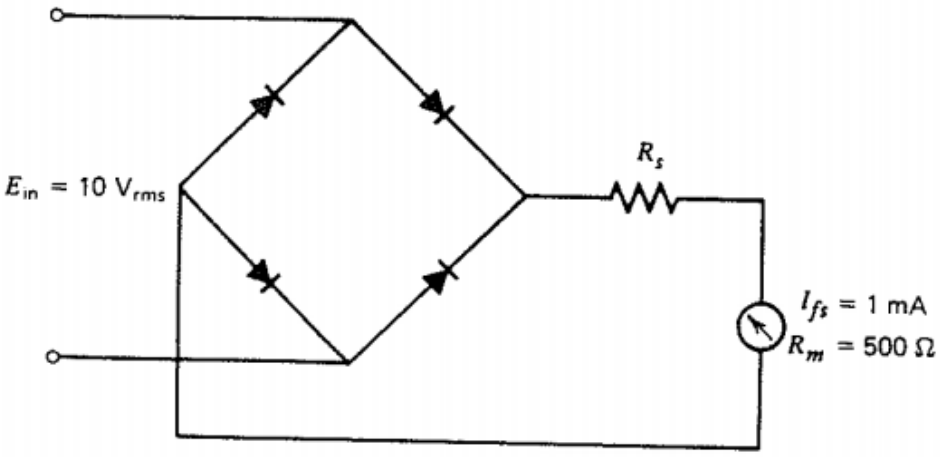


Figure 2-25 an AC voltmeter circuit that utilizes full-wave rectification.

The ac sensitivity is 90% of the dc sensitivity:

$$S_{ac} = 0.9S_{dc} = 0.9 \times \frac{1\ \text{k}\Omega}{\text{V}} = \frac{900\ \Omega}{\text{V}}$$

The multiplier resistor is therefore found to equal

$$R_s = S_{ac} \times \text{Range} - R_m$$
$$= \frac{900\ \Omega}{\text{V}} \times 10\ \text{V}_{rms} - 500\ \Omega = 8.5\ \text{k}\Omega$$

2-13 Electrodynamometer Movement

The electrodynamometer movement is one of the most fundamental and widely used meter movements today. Similar to the d'Arsonval movement, the electrodynamometer is a current-sensitive device, meaning the pointer moves up scale in response to current flow through a moving coil. Despite being a fundamental design, it is highly versatile.

Single-coil electrodynamometer movements can measure direct or alternating current or voltage, and can be used in single-phase wattmeters or varmeters.

One of the most significant applications of electrodynamometer movements is as standards for voltmeters and ammeters. Due to their inherent accuracy, electrodynamometer movements are ideal for use in standard instruments, which are used for calibrating other meters. The term "transfer instrument" refers to an instrument that is calibrated with a DC source and then used Without modification to measure AC current, providing a direct means of comparing AC and DC voltage and current measurements.

The single-coil electrodynamometer movement, as illustrated in Fig. 2-26, consists of a fixed coil split into two equal halves, with a movable coil positioned between them. Both halves of the split fixed coil and the moving coil are connected in series, and current from the measured circuit flows through all the coils, creating a magnetic field around the fixed coils. This field causes the movable coil to rotate.

Compared to a d'Arsonval movement, the basic electrodynamometer movement can handle much higher currents without requiring a shunt. It can manage currents of up to approximately 100 mA without shunt resistance. This capability is due to the design of the meter movement. The magnetic coupling between the fixed

and moving coils occurs across an air gap, resulting in a weaker magnetic field. To ensure sufficient magnetic coupling, more current must flow through the coils, necessitating the use of larger-diameter wire. Although this wire has lower resistance, it results in the electrodynamometer movement having a relatively low sensitivity rating, approximately 20 to 100 Ω/V.

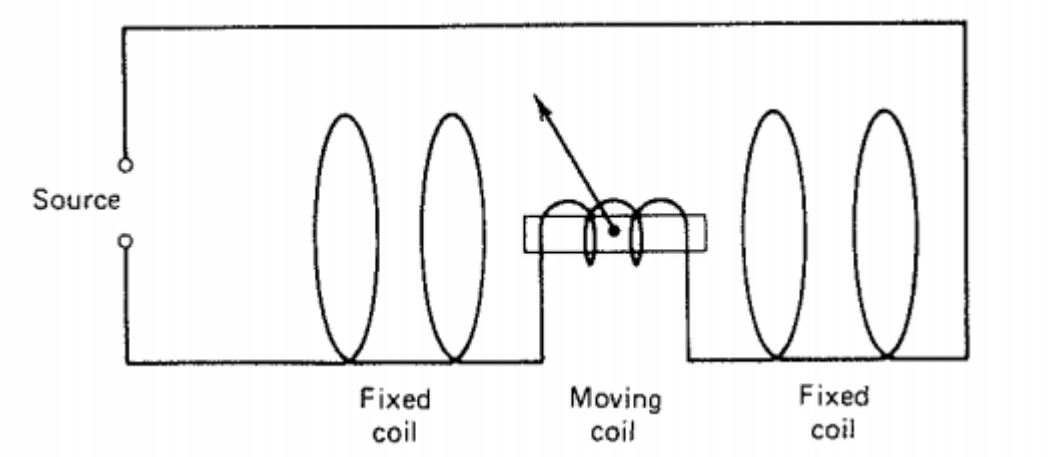


Figure 2-26 Electrodynamometer Movement

Example 2-15

An electrodynamometer movement with a full-scale deflection current rating of 10 mA in a voltmeter circuit is used, calculate the value of the multiplier needed for a 10-V range, given that R_m is 50 Ω.

The sensitivity of the meter movement is

$$S = \frac{1}{I_{fs}} = \frac{1}{10\text{ mA}} = \frac{100\ \Omega}{\text{V}}$$

Therefore, the value of the multiplier resistor is

$$R_s = S \times \text{Range} - R_m$$

$$= \frac{100\ \Omega}{\text{V}} \times 10\text{ V} - 50\ \Omega = 950\ \Omega$$

When expanding the current-measuring capabilities of an electrodynamometer movement by using a shunt resistor, the shunt is typically placed in parallel with only the moving coil, as illustrated in Fig. 2-27. Since only the moving coil is shunted, the resistance of the moving coil must be known in order to calculate the appropriate value for the shunt resistor.

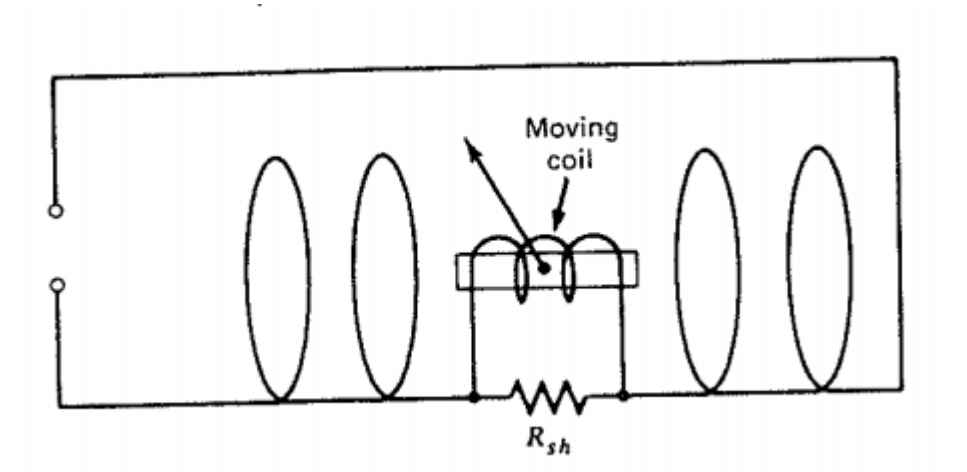


Figure 2-27 Electrodynamometer Movement used as ammeter

Example 2-16

An electrodynamometer movement with a full-scale deflection current rating of 10 mA is to be used as a 1-A ammeter. If the resistance of the moving coil is 40 Ω, what is the value of the shunt?

$$R_{sh} = \frac{R_m}{n-1}$$

$$= \frac{40\ \Omega}{100-1} = \frac{40\ \Omega}{99} = 0.404\ \Omega$$

If the ammeter described in Example 2-16 is connected to a 1-A DC source, the meter pointer should deflect to full scale. The same full-scale deflection should occur if the 1-A DC source is replaced with a 1-A RMS AC source.

Since the same current flows through both the field coils and the moving coil, whether the electrodynamometer movement is used as an ammeter or a voltmeter, the pointer deflection corresponds to the square of the current.

One of the most common applications for the electrodynamometer movement is in wattmeters. These wattmeters can measure both AC and DC power. AC signals are not limited to sinusoidal waveforms, allowing for the measurement of power developed by any AC waveform. When used as a wattmeter, the fixed coils, known as 'field' coils, are placed in series with the load and thus conduct the same current as the load (plus a small current through the moving coil).

2-14 Loading Effects Of AC Voltmeters

The sensitivity of AC voltmeters—whether using half-wave or full-wave rectification—is lower than that of DC voltmeters. Consequently, the loading effect of an AC voltmeter is greater than that of a DC voltmeter.

Example 2-17

Determine the reading obtained with a dc voltmeter in the circuit in Fig. 3-19 when switch S is set to position A; then set the switch to position B and determine the reading obtained with a half-wave and a full-wave ac voltmeter. All the meters use a 100-μA full-scale deflection meter movement and are set on their 10-V dc or rms ranges.

The reading obtained with the dc voltmeter

$$S_{dc} = \frac{1}{I_{fs}} = \frac{1}{100\ \mu A} = \frac{10\ k\Omega}{V}$$

$$R_s = S_{dc} \times \text{Range}$$

$$= \frac{10\ k\Omega}{V} \times \frac{10\ V}{1} = 100\ k\Omega$$

$$E = 20\ V \times \frac{100\ k\Omega \| 10\ k\Omega}{100\ k\Omega \| 10\ k\Omega + 10\ k\Omega}$$

$$= 20\ V \times \frac{9.09\ k\Omega}{9.09\ k\Omega + 10\ k\Omega} = 9.52\ V$$

The reading obtained with the ac voltmeter using half-wave rectification is computed as

$$S_{hw} = 0.45 S_{dc} = \frac{4.5\ \text{k}\Omega}{\text{V}}$$

$$R_s = S_{hw} \times \text{Range} = 45\ \text{k}\Omega$$

$$E = 20\ \text{V} \times \frac{45\ \text{k}\Omega \| 10\ \text{k}\Omega}{45\ \text{k}\Omega \| 10\ \text{k}\Omega + 10\ \text{k}\Omega}$$

$$= 20\ \text{V} \times \frac{8.18\ \text{k}\Omega}{8.18\ \text{k}\Omega + 10\ \text{k}\Omega} = 9.0\ \text{V}$$

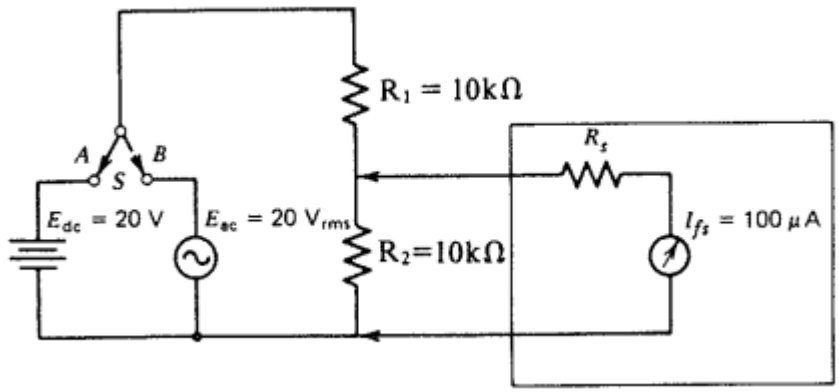
Finally, the reading obtained with the ac voltmeter using full-wave rectification is computed as

$$S_{fw} = 0.90 S_{dc} = \frac{9.0\ \text{k}\Omega}{\text{V}}$$

$$R_s = S_{fw} \times \text{Range} = 90\ \text{k}\Omega$$

$$E = 20\ \text{V} \times \frac{90\ \text{k}\Omega \| 10\ \text{k}\Omega}{90\ \text{k}\Omega \| 10\ \text{k}\Omega + 10\ \text{k}\Omega}$$

$$= 20\ \text{V} \times \frac{9\ \text{k}\Omega}{9\ \text{k}\Omega + 10\ \text{k}\Omega} = 9.47\ \text{V}$$



Chapter 3

Capacitance Measurement

3-1 Introduction

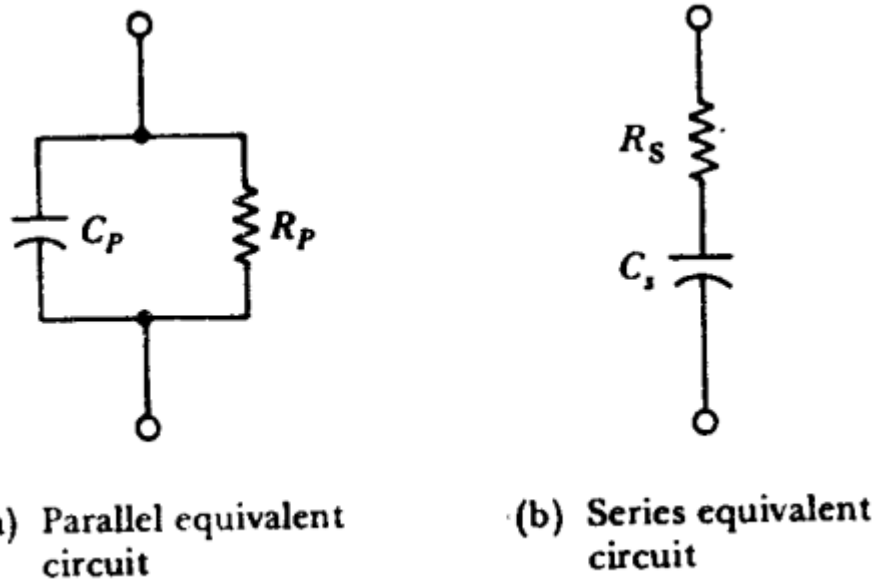
Inductance, capacitance, inductor Q factor, and capacitor D factor can all be accurately measured using AC bridges, which are modifications of the Wheatstone bridge. These measurements require an AC supply, and the null detector must be an AC instrument. A variety of AC bridge circuits are available for different specialized measurements. Some commercial AC bridges utilize only a standard capacitor and three adjustable standard resistors to create various types of inductance and capacitance bridge circuits. Special techniques are needed for measuring very small inductance and capacitance values. For digital measurement, inductance, capacitance, and resistance are first converted into a voltage using specific circuits. Capacitors and inductors designed to operate at high frequencies are most effectively measured with a Q meter.

3-2 RC and RL Equivalent Circuits

Capacitor Equivalent Circuits

The equivalent circuit of a capacitor consists of a pure capacitance Cp and a parallel resistance Rp, as shown in Figure 3-1(a). Here, Cp represents the actual capacitance value, while Rp accounts for the dielectric's resistance or leakage resistance. Capacitors with high leakage currents through the dielectric will have a relatively low Rp in their equivalent circuit. In contrast, very low leakage currents correspond to extremely high Rp values.

A parallel RC circuit has an equivalent series RC circuit, as shown in Figure 3-1(b). Either the series or parallel equivalent circuit can be used to represent a capacitor in a given circuit. Capacitors with high-resistance dielectrics are best represented by the series RC circuit, while those with low-resistance dielectrics are more accurately modeled by the parallel equivalent circuit. However, when measuring a capacitor in terms of its series capacitance and resistance, it is often useful to convert these values into their parallel equivalents.



(a) Parallel equivalent circuit

(b) Series equivalent circuit

Figure 3-1 The parallel and series equivalent circuit

Equivalent circuit quantities are often expressed in terms of their parallel equivalents because the (parallel) leakage resistance more accurately represents the quality of the capacitor's dielectric. The equations that connect the series and parallel equivalent circuits are derived below.

Referring to Figure 3-1, the series impedance is determined by:

$$Z_s = R_s - jX_s$$

and the parallel admittance is

$$Y_p = \frac{1}{R_p} + j\frac{1}{X_p} = G_p + jB_p$$

where G is conductance and B is susceptance. The impedances of each circuit must be equal.

Thus,
$$Z_s = \frac{1}{Y_p}$$

giving
$$R_s - jX_s = \frac{1}{G_p + jB_p}$$

or $$G_p + jB_p = \frac{1}{R_s - jX_s}$$

$$= \frac{1}{R_s - jX_s}\left(\frac{R_s + jX_s}{R_s + jX_s}\right)$$

giving $$G_p + jB_p = \frac{R_s + jX_s}{R_s^2 + X_s^2}$$

Equating the real terms,

$$G_p = \frac{R_s}{R_s^2 + X_s^2}$$

or $$\boxed{R_p = \frac{R_s^2 + X_s^2}{R_s}}$$

Equating the imaginary terms,

$$B_p = \frac{X_s}{R_s^2 + X_s^2}$$

or $$\boxed{X_p = \frac{R_s^2 + X_s^2}{X_s}}$$

3-3 D Factor of a Capacitor

The quality of a capacitor can be evaluated by its power dissipation. A capacitor with a very high dielectric resistance (low leakage current) will have minimal power dissipation, indicating a pure capacitance. Conversely, a capacitor with significant power dissipation and relatively low resistance (high leakage current) is considered lossy. The dissipation factor (D) quantifies the quality of the capacitor. The D factor is defined as the ratio of the component's reactance (at a specific frequency) to the resistance measurable at its terminals.

Using the parallel equivalent circuit:

$$D = \frac{X_p}{R_p} = \frac{1}{\omega C_p R_p}$$

Ideally, R_p should be very much larger than $1/(\omega C_p)$, giving a very small dissipation factor. Typically, D might range from 0.1 for electrolytic capacitors to less than 10^{-4} for capacitors with a plastic film dielectric (again depending on frequency).

For series equivalent circuit, D can be determined by:

$$D = \frac{R_s}{X_s} = \omega C_s R_s$$

Example 3-1

An unknown circuit exhibits the behaviour of a 0.005 µF capacitor in series with an 8 kΩ resistor when measured at a frequency of 1 kHz. An ohmmeter measures the terminal resistance as 134 kΩ. Determine the actual components of the circuit and the method of their connection.

$$X_s = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1\ \text{kHz} \times 0.005\ \mu\text{F}}$$

$$= 31.8\ \text{k}\Omega$$

$$R_s = 8\ \text{k}\Omega$$

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(8\ \text{k}\Omega)^2 + (31.8\ \text{k}\Omega)^2}{8\ \text{k}\Omega}$$

$$= 134\ \text{k}\Omega$$

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{(8\ \text{k}\Omega)^2 + (31.8\ \text{k}\Omega)^2}{31.8\ \text{k}\Omega}$$

$$= 33.8\ \text{k}\Omega$$

$$C_p = \frac{1}{2\pi f X_p} = \frac{1}{2\pi \times 1\ \text{kHz} \times 33.8\ \text{k}\Omega}$$

$$\approx 0.005\ \mu\text{F}$$

3-4 Capacitance Bridges

Simple Capacitance Bridge

The circuit for a simple capacitance bridge is shown in Figure 3-2. In this setup, Z_1 is a standard capacitor C_1, and Z_2 is the unknown capacitance C_x. Z_3 and Z_4 are known variable resistors, such as decade resistance boxes. When the bridge is balanced, the following relationship holds: $Z_1/Z_3 = Z_2/Z_4$.

$$Z_1 = \frac{-j1}{\omega C_1} \qquad Z_2 = \frac{-j1}{\omega C_x}$$

$$Z_3 = R_3 \qquad \text{and} \qquad Z_4 = R_4$$

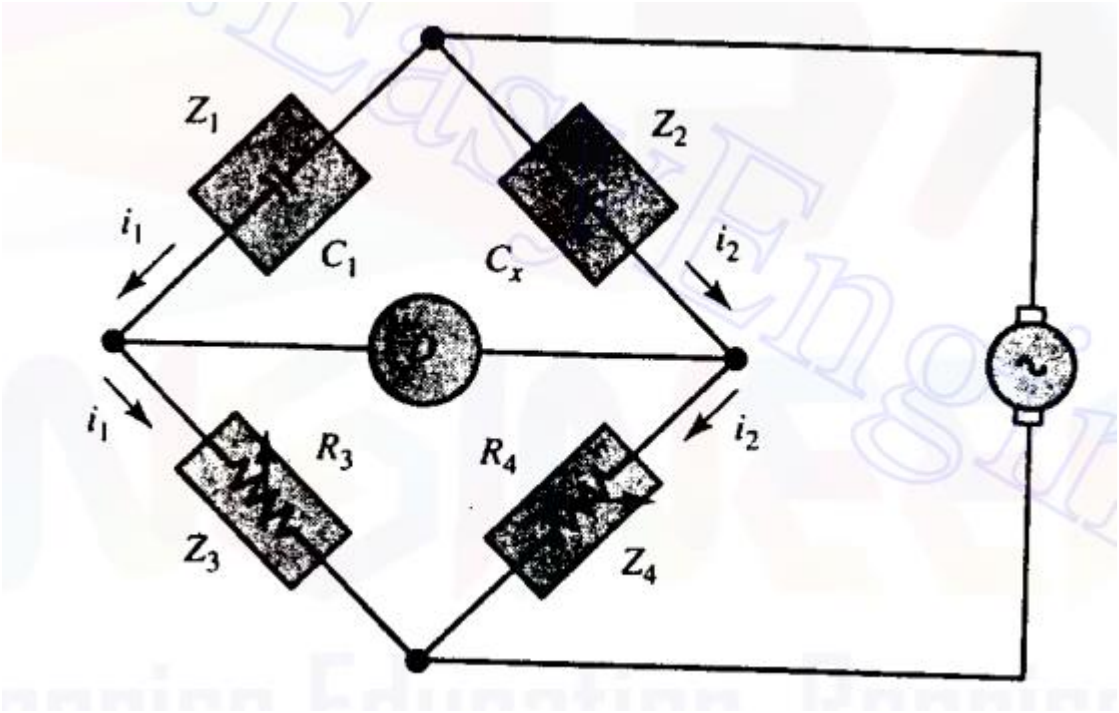


Figure 3-2 a simple capacitor bridge

$$\frac{-j1/\omega C_1}{R_3} = \frac{-j1/\omega C_x}{R_4}$$

$$\frac{1}{C_1 R_3} = \frac{1}{C_x R_4}$$

$$\boxed{C_x = \frac{C_1 R_3}{R_4}}$$

Example 3-2

the standard capacitance value in Figure 3-2 is C_1 =0.1 µF, and the ratio R_3/R_4 can be set anywhere between 100:1 and 1:100. Determine the range within which the unknown capacitance Cx can be measured.

$$C_x = \frac{C_1 R_3}{R_4}$$

For $R_3/R_4 = 100:1$:

$$C_x = 0.1\ \mu F \times \frac{100}{1}$$

$$= 10\ \mu F$$

For $R_3/R_4 = 1:100$:

$$C_x = 0.1\ \mu F \times \frac{1}{100}$$

$$= 0.001\ \mu F$$

The previous analysis of the simple capacitance bridge assumes that the capacitors are ideal, with no leakage current through the dielectric. If a resistance were added in series or parallel with the capacitor C_x while keeping the other bridge components unchanged, it would become nearly impossible to achieve balance. This difficulty arises because currents i_1 and i_2 could not be brought into phase, causing i_1R_3 and i_2R_4 to also be out of phase. the equivalent circuit of a leaky capacitor consists of a pure capacitance in parallel with a pure resistance. Therefore, the simple capacitance bridge is only suitable for measuring capacitors with high-resistance dielectrics.

3-5 Series-Resistance Capacitance Bridge

In the circuit depicted in Figure 3-3(a), the unknown capacitance is modeled as a pure capacitance Cs in series with a resistance Rs. A standard adjustable resistance R_1 is connected in series with a standard capacitor C_1. When the bridge is balanced, the voltage drop across R_1 compensates for the resistive voltage drops in branch Z_2. The resistor added in series with Cs increases the total resistive component in that branch, eliminating the need for inconveniently small values of R_1 to achieve balance. Achieving bridge balance is easier when each capacitive branch includes a significant resistive component. To obtain balance, R_1 and either R_3 or R_4 are alternately adjusted. The series-resistance capacitance bridge is particularly suitable for capacitors with high-resistance dielectrics, which exhibit very low leakage current and a low dissipation factor. When the bridge is balanced:

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\boxed{\frac{R_1 - j1/\omega C_1}{R_3} = \frac{R_s - j1/\omega C_s}{R_4}}$$

Equating the real terms

$$\frac{R_1}{R_3} = \frac{R_s}{R_4}$$

$$\boxed{R_s = \frac{R_1 R_4}{R_3}}$$

Equating the imaginary terms

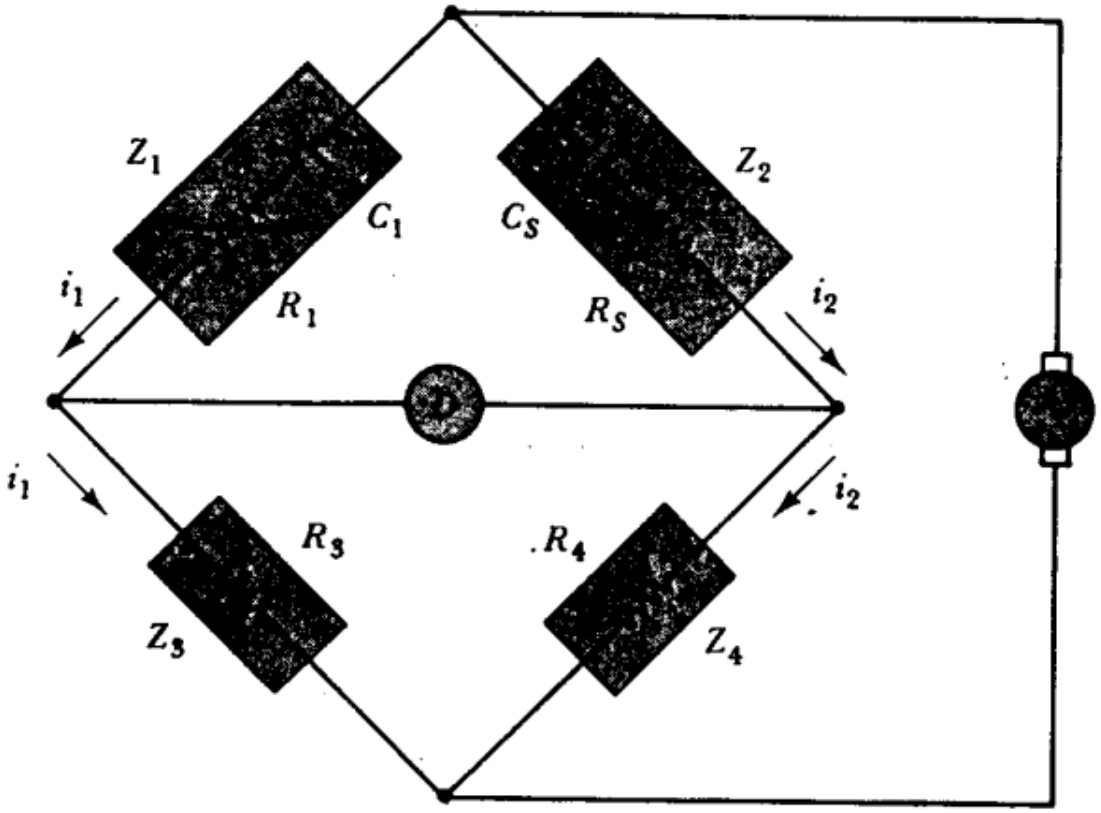
$$\frac{1}{\omega C_1 R_3} = \frac{1}{\omega C_s R_4}$$

$$\boxed{C_s = \frac{C_1 R_3}{R_4}}$$

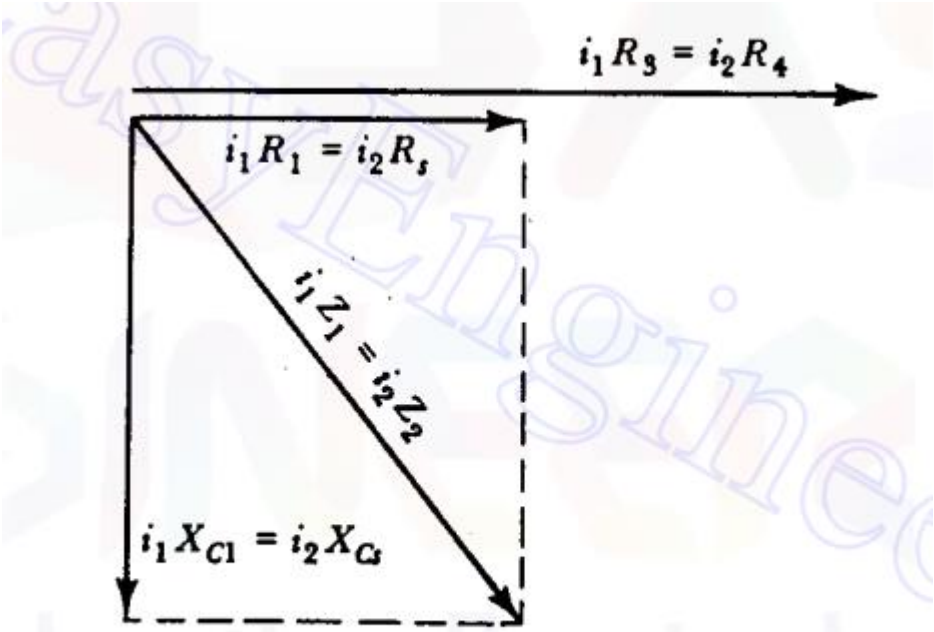
The phasor diagram for the series-resistance capacitance bridge at balance is shown in Figure 3-3(b). The voltage drops across Z_3 and Z_4 are i_1R_3 and i_2R_4, respectively. For the bridge to be balanced, these two voltages must be equal and in phase. In the phasor diagram, they are depicted as equal and in phase. Since R_3

and R_4 are resistive, i_1 is in phase with i_1R_3, and i_2 is in phase with i_2R_4. The impedance of C_1 is purely capacitive, causing the current to lead the voltage by 90° in a pure capacitance. Consequently, the voltage drop across the capacitor is i_1X_{C1} across C_1 is represented as lagging i_1 by 90° in the phasor diagram. Similarly, the voltage drop i_2X_{Cs} across Cs is shown as lagging i_2 by 90°. The resistive voltage drops i_1R_1 and i_2Rs are in phase with i_1 and i_2, respectively.

The total voltage drop across Z_1 is the phasor sum of i_1R_1 and i_1X_{C1}. Likewise, i_2Z_2 is the phasor sum of i_2Rs and i_2X_{Cs}. Since i_2Z_2 must be equal to and in phase with i_1Z_1, it follows that i_1R_1 is equal to i_2R_s, and i_1X_{C1} is equal to i_2X_{Cs}.



(a) Circuit of series-resistance capacitance bridge



(b) Phasor diagram for balanced bridge

Figure 3-3 Series-Resistance Capacitance Bridge and phasor diagram

Example 3-3

In a series-resistance capacitance bridge, as shown in Figure 3-3(a), a 0.4 μF standard capacitor C_1 is used, and R_3 = 10 kOhm. Balance is achieved at a supply frequency of 100 Hz when R_1 = 125 Ohm and R_4 = 14.7 kOhm. Calculate the resistive and capacitive components of the measured capacitor, as well as its dissipation factor.

$$C_s = \frac{C_1 R_3}{R_4} = \frac{0.1\ \mu F \times 10\ k\Omega}{14.7\ k\Omega}$$

$$= 0.068\ \mu F$$

$$R_s = \frac{R_1 R_4}{R_3} = \frac{125\ \Omega \times 14.7\ k\Omega}{10\ k\Omega}$$

$$= 183.8\ \Omega$$

$$D = \omega C_s R_s$$

$$= 2\pi \times 100\ Hz \times 0.068\ \mu F \times 183.8\ \Omega$$

$$\approx 0.008$$

3-6 Parallel-Resistance Capacitance Bridge

The circuit for a parallel-resistance capacitance bridge is shown in Figure 3-4. In this configuration, the unknown capacitance is represented by its parallel equivalent circuit, with Cp in parallel with Rp. Resistors Z_3 and Z_4 are included, as before, with one or both being adjustable. Z_2 is balanced by a standard capacitor C_1 in parallel with an adjustable resistor R_1. To achieve bridge balance, adjustments are made to R_1 and either R_3 or R_4. The parallel-resistance capacitance bridge is particularly suitable for capacitors with low-resistance dielectrics, which exhibit relatively high leakage current and a high dissipation factor. When the bridge is balanced:

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\frac{1}{Z_1} = \frac{1}{R_1} - \frac{1}{j(1/\omega C_1)}$$

$$= \frac{1}{R_1} + j\omega C_1$$

$$Z_1 = \frac{1}{1/R_1 + j\omega C_1}$$

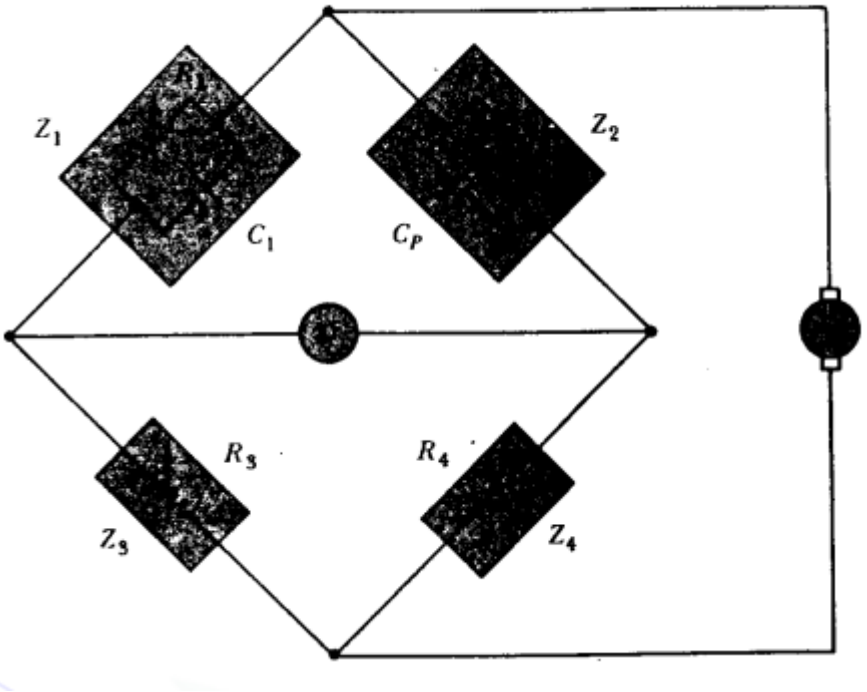


Figure 3-4 a parallel-resistance capacitance bridge

$$\frac{1}{Z_2} = \frac{1}{R_p} + \frac{1}{j(1/\omega C_p)}$$

$$= \frac{1}{R_p} + j\omega C_p$$

$$Z_2 = \frac{1}{1/R_p + j\omega C_p}$$

$$\frac{1/(1/R_1 + j\omega C_1)}{R_3} = \frac{1/(1/R_p + j\omega C_p)}{R_4}$$

$$\frac{1}{R_3(1/R_1 + j\omega C_1)} = \frac{1}{R_4(1/R_p + j\omega C_p)}$$

$$\boxed{R_3\left(\frac{1}{R_1} + j\omega C_1\right) = R_4\left(\frac{1}{R_p} + j\omega C_p\right)}$$

Equating the real terms

$$\frac{R_3}{R_1} = \frac{R_4}{R_p}$$

$$\boxed{R_p = \frac{R_1 R_4}{R_3}}$$

Equating the imaginary terms

$$\omega C_1 R_3 = \omega C_p R_4$$

$$\boxed{C_p = \frac{C_1 R_3}{R_4}}$$

Example 3-4

In a parallel-resistance capacitance bridge, as shown in Figure 3-4, the standard capacitance C_1 is 0.1 μF, and R_3 =10 kOhm. Balance is achieved at a supply frequency of 100 Hz when R_1 = 375 kOhm, R_3 = 10 kOhm, and R_4 = 14.7 kOhm.

Calculate the resistive and capacitive components of the measured capacitor, as well as its dissipation factor.

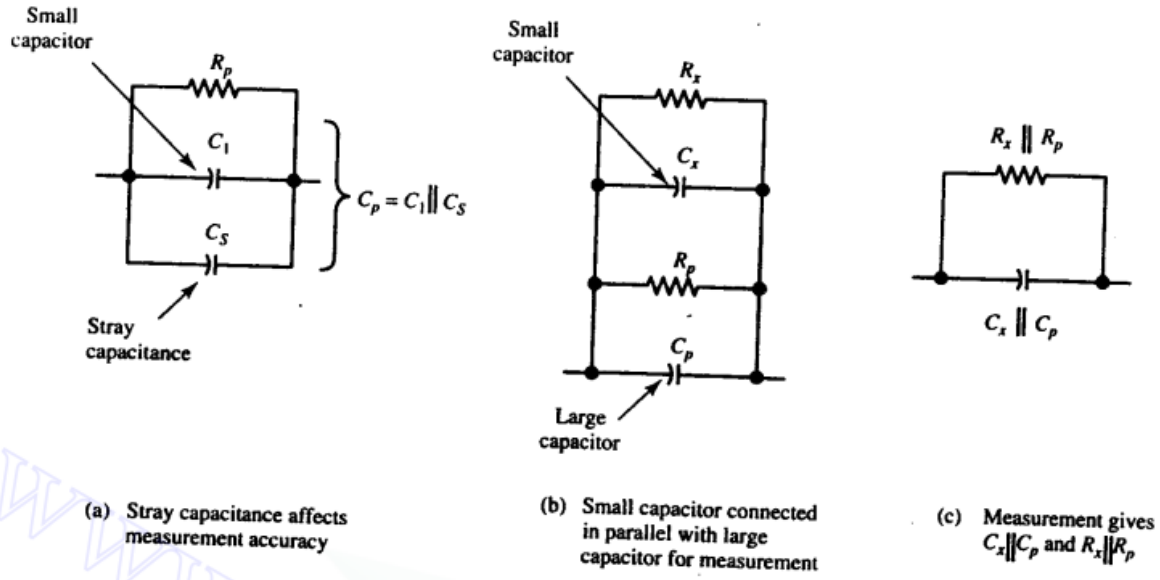
$$C_p = \frac{C_1 R_3}{R_4} = \frac{0.1\ \mu F \times 10\ k\Omega}{14.7\ k\Omega}$$
$$= 0.068\ \mu F$$
$$R_p = \frac{R_1 R_4}{R_3} = \frac{375\ k\Omega \times 14.7\ k\Omega}{10\ k\Omega}$$
$$= 551.3\ k\Omega$$
$$D = \frac{1}{\omega C_p R_p} = \frac{1}{2\pi \times 100\ Hz \times 0.068\ \mu F \times 551.3\ k\Omega}$$
$$= 42.5 \times 10^{-3}$$

3-7 Measuring Small C, R, and L Quantities

When measuring very small values of stray capacitance (C), inductance (L), or resistance (R), of connecting leads can cause significant errors. These errors can be minimized by connecting the unknown component directly to the bridge terminal or using very short connecting leads. However, even with these precautions, small internal L, C, and R values, known as residuals, exist within all instruments. Manufacturers typically specify these residuals, which for a typical impedance bridge might be R = 1×10^{-3} Ohm, C = 0.5 pF, and L = 0.2 µH. These residuals can introduce substantial errors if they are a significant percentage of the quantity being measured.

The errors caused by stray and residual values can be eliminated using a substitution technique, as illustrated in Figure 3-5. For capacitance measurements, the bridge is first balanced with a larger capacitor in place of the small capacitor to be measured. Then, the small capacitor is connected in parallel with the larger one, and the bridge is readjusted for balance. The first measurement gives the combined value of the large capacitance C_l and the stray and residual capacitance Cs, so the measured capacitance is C_l + Cs. When the small capacitor Cx is added, the total measured capacitance becomes C_l + Cs + Cx. The value of Cx is found by subtracting the first measurement from the second.

A similar technique is used for measuring small inductance and resistance values, except that in case the low-value component is connected in series with a larger L or R quantity. This substitution method can also be applied to other, non-bridge measurement techniques.



(a) Stray capacitance affects measurement accuracy

(b) Small capacitor connected in parallel with large capacitor for measurement

(c) Measurement gives $C_x \| C_p$ and $R_x \| R_p$

Figure 3-5 a substitution technique

Example 3-5

In the bridge, a new balance is achieved when a small capacitor Cx is connected in parallel with the measured capacitor Cp. The new component values for balance are R_1= 369.3 kOhm, R_3= 10 kOhm, and R_4= 14.66 kOhm. Determine the value of Cx and its parallel resistive component Rx.

$$C_x \| C_p = C_x + C_p = \frac{C_1 R_3}{R_4}$$

$$= \frac{0.1\ \mu\text{F} \times 10\ \text{k}\Omega}{14.66\ \text{k}\Omega} = 0.682\ \mu\text{F}$$

$$C_x = 0.0682\ \mu\text{F} - C_p = 0.0682\ \mu\text{F} - 0.068\ \mu\text{F}$$

$$= 200\ \text{pF}$$

and

$$R_x \| R_p = \frac{R_1 R_4}{R_3} = \frac{369.3\ \text{k}\Omega \times 14.66\ \text{k}\Omega}{10\ \text{k}\Omega}$$

$$= 541.4\ \text{k}\Omega$$

$$\frac{1}{R_x} + \frac{1}{R_p} = \frac{1}{R_x \| R_p}$$

$$R_x = \frac{1}{1/(R_x \| R_p) - 1/R_p}$$

$$R_p = 553.1\ \text{k}\Omega$$

$$R_x = \frac{1}{1/541.4\ \text{k}\Omega - 1/553.1\ \text{k}\Omega}$$

$$= 30\ \text{M}\Omega$$

3-8 Digital C Measurement

Capacitive impedance is handled similarly to inductive impedance. However, in this case, the input voltage is applied across the capacitor, and the output voltage is measured across the resistor (see figure 3-6).

In this case $I = V_i/X_c$, and $V_R = IR$. With $V_i = 1.592$ Vrms, $f = 1$ kHz, $R_1 = 1\ \text{k}\Omega$, and $C = 0.1\ \mu\text{F}$:

$$I = \frac{V_i}{X_c} = V_i(2\pi fC)$$

$$= 1.592\ \text{V} \times 2\pi \times 1\ \text{kHz} \times 0.1\ \mu\text{F}$$

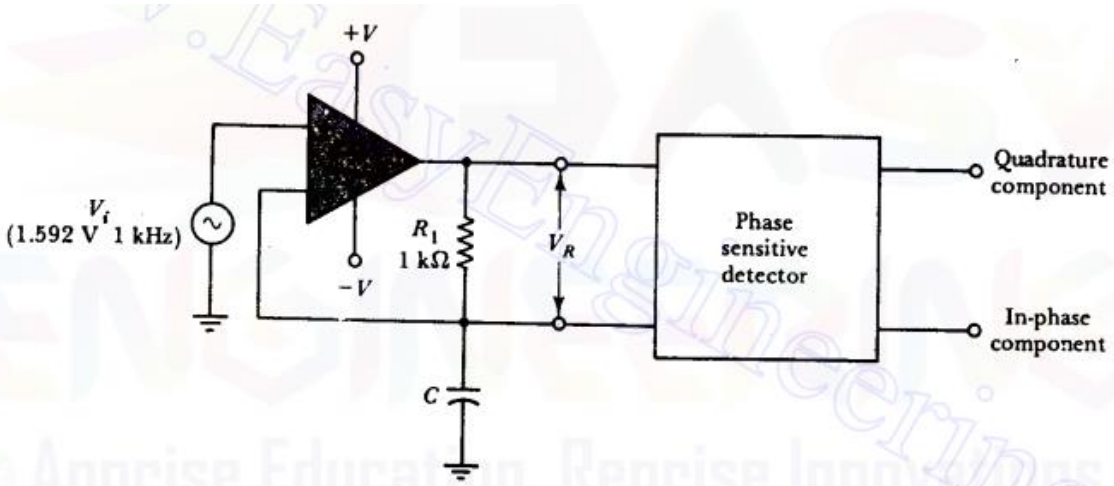


Figure 3-6 linear conversion of capacitive impedance into voltage

$$= 1\ \text{mA}$$

and

$$V_R = IR = 1\ \text{mA} \times 1\ \text{k}\Omega$$

$$= 1\ \text{V (rms)}$$

when $C = 0.2\ \mu\text{F}$, $V_R = 2$ V; when $C = 0.3\ \mu\text{F}$, $V_R = 3$ V; and so on.

CHAPTER FOUR

Cathode-Ray Oscilloscope

4-1 Introduction

An oscilloscope is a fundamental instrument used for studying a wide range of waveforms. It can measure various quantities such as voltage, phase difference, pulse width, delay time, rise time, and fall time. The oscilloscope consists of a cathode-ray tube (CRT) along with its associated control and input circuitry. In the CRT, electrons emitted from a heated cathode are formed into a fine beam and accelerated toward a fluorescent screen. The screen emits light at the point where the electrons hit. The electron beam can be easily deflected both vertically and horizontally by applying voltages to deflection plates. Typically, the beam is swept horizontally across the screen by a ramp voltage produced by a time base circuit, while the waveform under investigation is used to deflect the beam vertically. This process results in a voltage-versus-time display of the input waveform. Most oscilloscopes are dual-trace (or multi-trace) instruments, allowing them to display two or more waveforms simultaneously.

4-2 Cathode-Ray Tube

Operation:

The basic construction and operation of a cathode-ray tube (CRT) are depicted in Figure 4-1. The CRT consists of a system of electrodes housed in an evacuated glass tube with a viewing screen at one end. A beam of electrons is produced by the cathode and directed toward the screen, where it causes the phosphor coating to emit light at the point of impact. The electron beam can be deflected both vertically and horizontally by externally applied voltages.

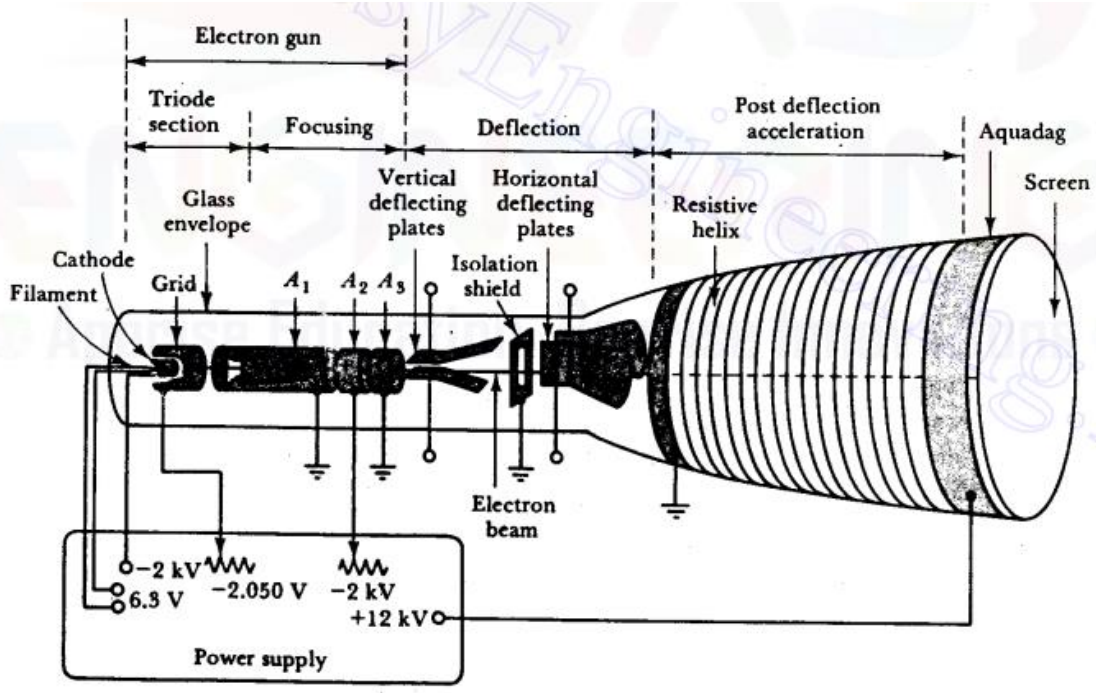


Figure 4-1 a cathode-ray tube (CRT)

Triode Section

The triode section of the tube includes a cathode, grid, and anode. The grid, a nickel cup with a hole, almost completely surrounds the cathode. The cathode, also made of nickel, is cylindrical with a flat, oxide-coated surface that emits electrons toward the grid's hole. An internal filament heats the cathode, which is typically at around -2 kV, while the grid's potential can be adjusted from -2000 V to -2050 V. The potential difference between the grid and cathode regulates the flow of electrons from the cathode, thereby controlling the number of electrons that reach the screen. More electrons result in a brighter glow on the screen, while fewer electrons produce a dimmer glow, making the grid potential a brightness control.

The first anode (A_1) is cylindrical, closed at one end with a hole in the center. A_1 is grounded, making it highly positive relative to the negatively charged cathode. This positive potential accelerates the electrons from the cathode through the holes in the grid and anode toward the tube's focusing section.

Focusing Section

The focusing electrodes A_1, A_2, and A_3 function as an electron lens, concentrating the electron beam to a fine point on the screen. A_1 generates the accelerating field that draws electrons from the cathode, with its hole restricting the beam's initial cross-section. Both A_3 and A_1 are grounded, while the potential of A_2 can be adjusted around -2 kV. This potential difference between the anodes creates

equipotential lines, where the voltage remains constant along each line. While the lines within A_3 produce a diverging force. By adjusting the potential on A_2, which is often called the focus ring, the point at which the beam is focused can be fine-tuned.

The negative potential on A_2 slows the electrons, but they are re-accelerated by A_3, resulting in the electron beam maintaining a constant velocity as it passes between the deflecting plates.

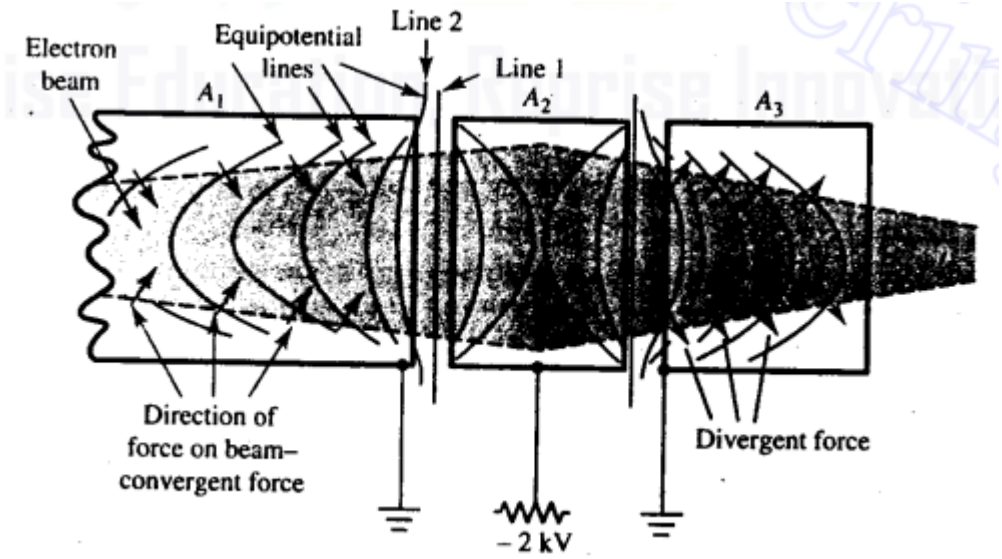


Figure 4-2 The electrostatic focusing of cathode-ray tube

Deflection Section

If the horizontal and vertical deflecting plates are grounded or left unconnected, the electron beam would pass between each pair of plates and strike the center of the oscilloscope screen, creating a bright, glowing point. When a positive voltage is applied to one plate of a pair of deflecting plates and a negative potential to the other, the electrons in the beam are attracted to the positive plate and repelled by the negative plate. This causes the electrons to accelerate toward the positive plate. However, since the electrons are traveling along the axis between the plates, they don't hit the plates themselves. Instead, the beam is deflected, causing the electrons to strike the screen at a new position.

Screen

The screen of a CRT is coated with phosphor materials on the inside of the tube face. When the electron beam strikes the screen, the phosphor's electrons are excited to a higher energy level and emit light as they return to their normal state. The glow can persist for varying durations, from a few milliseconds to several

seconds or longer, depending on the phosphor material used. The color of the glow can be blue, red, green, or white.

Phosphors are insulators, so without a mechanism to counteract it, a negative charge would accumulate on the screen, eventually repelling the electron beam. This is prevented by secondary emission, where secondary electrons are collected by a graphite coating, called aquadag, around the neck of the tube. In some CRTs, an aluminum film is deposited on the screen's surface. This film allows the electron beam to pass through while collecting secondary electrons and grounding them. Additionally, the aluminum film improves brightness by reflecting emitted light back toward the glass and serves as a heat sink to protect the screen.

Brightness of display

The brightness of the screen's glow depends on the number of electrons in the beam, which is controlled by the grid voltage, making it a brightness control. Brightness also increases with the speed of the electron beam. However, high-speed electrons passing through the deflection plates reduce the deflection sensitivity, so post-deflection acceleration is used. A resistive helix is deposited inside the tube from the deflecting plates to the screen, providing a continuous accelerating force to the electrons leaving the deflecting plates, ensuring they strike the screen with high energy.

Deflection Amplifiers

To deflect the electron beam, the input voltage must be converted into two equal and opposite voltages, +E/2 and -E/2. This requires an amplifier that can take an AC or DC input and produce differential outputs.

4-3 Waveform Display

When an alternating voltage is applied to the vertical deflecting plates without any input to the horizontal plates, the spot on the tube face moves up and down continuously. However, when a constantly increasing voltage is applied to the horizontal deflecting plates, the spot moves horizontally.

Consider Figure 4-3, where a sine wave is applied to the vertical deflecting plates, and a sawtooth (or repetitive ramp) voltage is applied to the horizontal plates. If these waveforms are perfectly synchronized, at time t=0, the vertical deflecting voltage is zero, and the horizontal deflecting voltage is -2 V. Assuming a deflection sensitivity of 2 cm/V, the vertical deflection is zero, and the horizontal deflection is 4 cm to the left of the screen's center (point 1 in Figure 4-3(c)).

At t=0.5 ms, the horizontal deflecting voltage changes to -1.5 V, resulting in a horizontal deflection of 3 cm left of the screen center. Simultaneously, the vertical deflecting voltage increases to +1.4 V, causing a vertical deflection of +2.8 cm above the screen center. The spot is now positioned 2.8 cm up and 3 cm left of the screen center, marked as point 2 in Figure 4-3(c).

The table following this description provides deflection voltages and corresponding deflections at various time intervals for the waveforms shown in Figure 4-3.

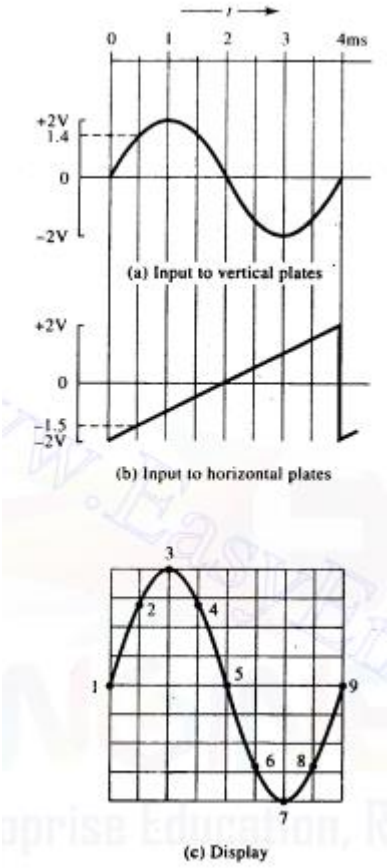


Figure 4-3 a ramp waveform is applied to the horizontal deflecting plates

t (ms)	1	1.5	2	2.5	3	3.5	4
Vertical voltage (V)	+2	+1.4	0	−1.4	−2	−1.4	0
Vertical deflection (cm)	+4	+2.8	0	−2.8	−4	−2.8	0
Horizontal voltage (V)	−1	−0.5	0	+0.5	+1	+1.5	+2
Horizontal deflection (cm)	−2	−1	0	+1	+2	+3	+4
Point	3	4	5	6	7	8	9

Example 4-1

A 500 Hz triangular wave with a peak amplitude of 40 V is applied to the vertical deflecting plates of a CRT. A 250 Hz sawtooth wave with an amplitude of 50 V

is applied to the horizontal deflecting plates. The CRT has a vertical deflection sensitivity of 0.1 cm/V and a horizontal deflection sensitivity of 0.08 cm/V. Assuming the inputs are synchronized, determine the waveform displayed on the screen.

For the triangular wave,

$$T = \frac{1}{f} = \frac{1}{500\ \text{Hz}} = 2\ \text{ms}$$

For the sawtooth wave,

$$T = \frac{1}{250\ \text{Hz}} = 4\ \text{ms}$$

The two waveforms are shown in Figure 9-6(a) and (b).
At t = 0:

vertical voltage = 0

horizontal voltage = −50 V

horizontal deflection = voltage × (deflection sensitivity)

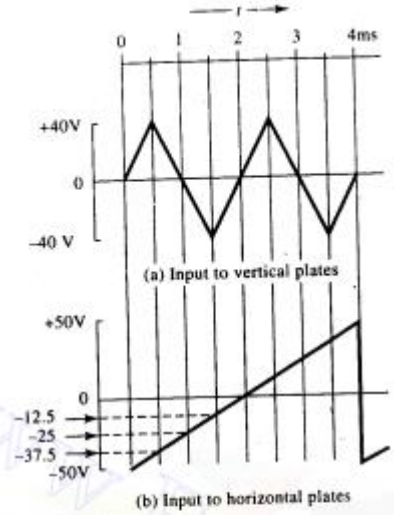
= −50 × 0.08 cm

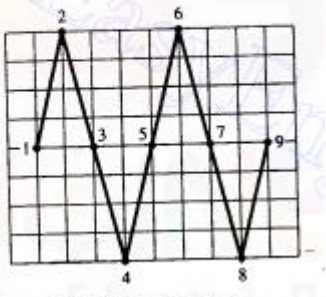
= −4 cm (i.e., 4 cm left from center)

Point 1 on the CRT screen [Figure 9-6(c)] is at

vertical deflection = 0,

horizontal deflection = 4 cm left of center





At t = 0.5 ms:

vertical voltage = +40 V

horizontal voltage = −37.5 V

Therefore, at point 2 on the CRT screen,

vertical deflection = +40 × 0.1 cm = +4 cm

horizontal deflection = −37.5 × 0.08 cm = −3 cm

At t = 1 ms (point 3):

vertical deflection = 0

horizontal deflection = −25 × 0.08 cm = −2 cm

At t = 1.5 ms (point 4):

vertical deflection = −40 × 0.1 cm = −4 cm

horizontal deflection = −12.5 × 0.08 cm = −1 cm

At t = 2 ms (point 5):

vertical deflection = 0

horizontal deflection = 0

t *(ms)*	2.5	3	3.5	4
Vertical voltage *(V)*	+40	0	−40	0
Vertical deflection *(cm)*	+4	0	−4	0
Horizontal voltage *(V)*	+12.5	+25	+37.5	+50
Horizontal deflection *(cm)*	+1	+2	+3	+4
Point	6	7	8	9

9-4 Oscilloscope Controls

The front panel of a representative single-beam dual-trace analog oscilloscope, as shown in Figure 4-4, includes several essential controls for operating the instrument. Here's an overview of these controls and their functions:

- Screen and Graticule: The waveforms are displayed on the screen, which is covered by a calibrated flat plastic sheet called a graticule. The graticule helps measure the amplitude (in vertical divisions) and the time period (in horizontal divisions) of the displayed waveforms.

- Power and Display Controls:

  - POWER ON/OFF: Switches the oscilloscope on or off.

  - INTENSITY: Adjusts the brightness of the waveform display.

  - FOCUS: Sharpens the waveform to ensure a clear, fine line.

  - BEAM FINDER: Locates the display if it has shifted off the screen.

- Vertical Controls: For displaying two waveforms, there are separate sets of vertical controls for each channel:

- CHANNEL A and CHANNEL B: These controls manage the vertical position and sensitivity of each waveform.

- POSITION Controls: Adjusts the vertical position of each waveform on the screen for optimal viewing.

- VOLTS/DIV Selector Switch: Sets the sensitivity of the display to input voltages. For example, if set to 1 V, a signal with a peak-to-peak amplitude of 1 V will span 1 division on the screen. A signal occupying four divisions at this setting indicates a 4 V peak-to-peak amplitude. The setting is accurate only when the vernier knob is in the CAL (calibrated) position.

- Vernier Control: Provides fine adjustment of the VOLTS/DIV setting, allowing for more precise voltage scaling, typically reducing the display amplitude by a factor of 2.5.

These controls work together to ensure that the oscilloscope displays waveforms accurately and clearly, allowing for effective measurement and analysis of electronic signals.

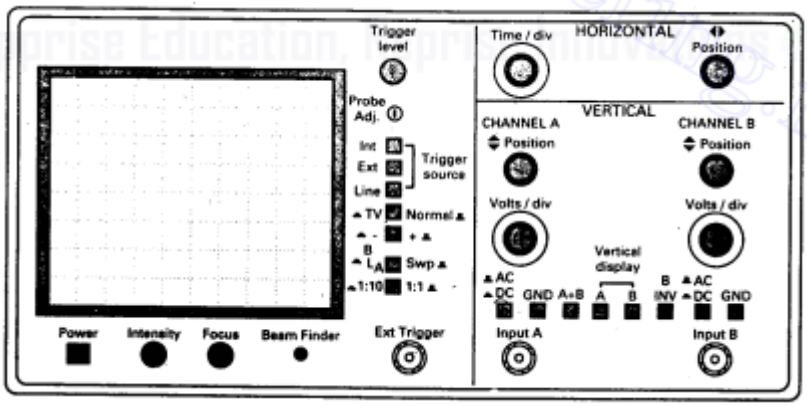


Figure 4-4 single-beam dual-trace analog oscilloscope

Below the VOLTS/DIV switches for Channel A and Channel B, there is a row of pushbutton switches, with Channel A and Channel B input terminals (coaxial cable) located beneath these switches. The pushbuttons directly above each input terminal allow for AC or DC coupling of the input voltage:

- AC-DC Button: Setting this to AC enables a coupling capacitor to pass only the AC component of the signal while blocking the DC component. This is useful for displaying an alternating voltage while ignoring any superimposed DC level.

- GND (Ground) Button: This button disconnects the input signal and grounds the input terminal, allowing you to set each trace to a convenient zero position on

the screen. The DC level of the input voltage can then be measured relative to this zero position.

Additionally, the VERTICAL DISPLAY A and B pushbuttons allow you to choose which channels to display on the screen.

The B INV button inverts Channel B, while the A + B button enables the addition of Channel A and Channel B waveforms, displaying their combined result. If both the A + B and B INV buttons are pressed, the displayed waveform represents the difference between Channel A and Channel B input voltages.

The HORIZONTAL TIME/DIV selector switch determines how many horizontal divisions each cycle of the displayed waveform occupies. This control, also known as the time base capacitor selection switch (S1) as shown in Figure 4-4, affects both waveforms. For example, with the TIME/DIV switch set to 1 ms, a waveform cycle occupying 1 horizontal division corresponds to a time period of 1 ms. If the waveform cycle spans 3.5 divisions at 1 ms/div, its time period is 3.5 ms. The TIME/DIV setting is accurate only when the vernier knob is in the CAL (calibrated) position. The vernier control allows for continuous adjustment, expanding the display of one cycle of the waveform up to 10 times the horizontal time/div setting.

The HORIZONTAL POSITION knob moves the displayed waveforms horizontally across the screen, similar to how the VERTICAL POSITION controls adjust vertical placement.

The TRIGGER LEVEL knob adjusts the point at which the input waveform triggers the sweep. When set to AUTO OFF, the time base circuit stops operating automatically, requiring each horizontal sweep of the electron beam to be triggered by an input waveform.

Below the TRIGGER LEVEL control is the PROBE ADJ (Probe Adjust) terminal. This terminal provides a 2 kHz, 0.5 V square-wave output, which is used for calibrating attenuator probes.

The first three pushbutton switches in the vertical row below the PROBE ADJ terminal are used to select the trigger source and operate as follows:

- INT (Internal): Selects the internal trigger source, which means the time base is triggered by one of the input waveforms. When both Channel A and Channel B are displayed, Channel A is used as the internal trigger source.

- EXT (External): Triggers the time base from an external source connected to the EXT TRIGGER terminal.

- LINE: Triggers the time base from the line or AC power frequency.

The TV-NORM pushbutton is typically set to the NORM position for general use. For analyzing TV video signals, switch it to the TV position. The — + button allows you to choose whether the waveform starts at the beginning of the positive or negative half-cycle, performing the same function as switch S2.

The A-B SWP button can be set to either the SWP (sweep) or A-B position:

- SWP: The internal time base sweeps the electron beam horizontally across the screen for waveform display.

- A-B: Channel B inputs produce vertical deflection, while Channel A inputs provide horizontal deflection. This mode is useful for displaying the characteristics of electronic devices.

4-5 Measurement of Voltage, Frequency, and Phase

Peak-to-Peak Voltage Measurement:

To measure the peak-to-peak amplitude of a displayed waveform on an oscilloscope, use the graticule divisions on the screen. For instance, Figure 4-5 shows two sine waves with different amplitudes and time periods. Waveform A has a peak-to-peak amplitude of 4.6 vertical divisions, while Waveform B measures 2 vertical divisions. Ensure that the central vernier knob on the VOLTS/DIV control is set to the CAL (calibrated) position before measuring the waveform's amplitude.

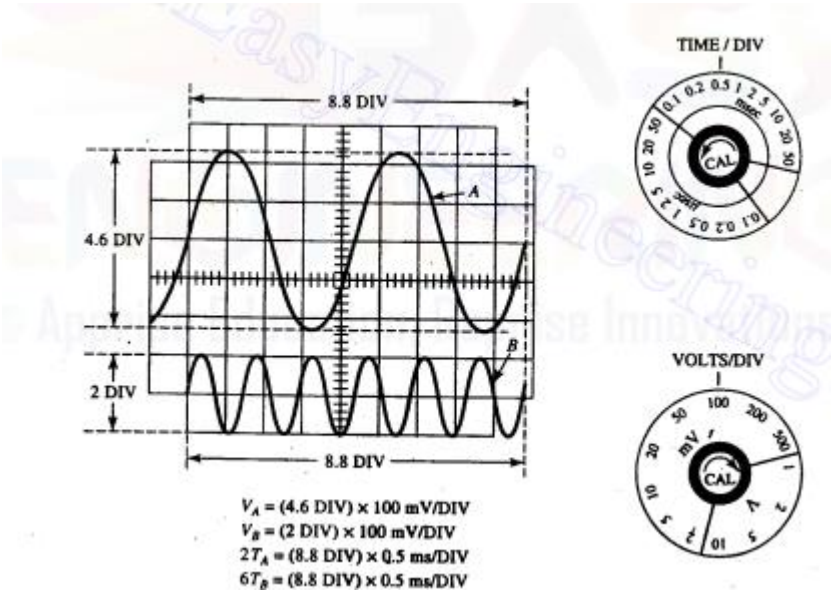


Figure 4-5 two sine waves with different amplitudes and time periods

$$V_{p\text{-}to\text{-}p} = (\text{vertical p-to-p divisions}) \times (\text{VOLTS/DIV})$$

Frequency Determination:

To determine the time period of a sine wave, measure the time for one complete cycle in horizontal divisions on the oscilloscope screen and multiply by the TIME/DIV setting:

$$T = (\text{horizontal divisions/cycle}) \times (\text{TIME/DIV})$$

The frequency is then calculated as the reciprocal of the time period. Before measuring the time period, ensure that the central vernier knob on the TIME/DIV control is set to the CAL (calibrated) position. For example, if the TIME/DIV control is set to 0.5 ms, you can calculate the time period and frequency of each wave as shown in Figure 4-5.

Wave *A*,

$$T = \frac{(8.8 \text{ divisions}) \times 0.5 \text{ ms}}{2 \text{ cycles}} = 2.2 \text{ ms}$$

$$f = \frac{1}{2.2 \text{ ms}} \simeq 455 \text{ Hz}$$

Wave *B*,

$$T = \frac{(8.8 \text{ divisions}) \times 0.5 \text{ ms}}{6 \text{ cycles}} = 0.73 \text{ ms}$$

$$f = \frac{1}{0.73 \text{ ms}} \simeq 1.36 \text{ kHz}$$

Phase Measurement:

To measure the phase difference between two waveforms, refer to Figure 4-6. If each waveform has a time period of 8 horizontal divisions and the time between the start of each cycle is 1.4 divisions, you can calculate the phase difference as follows:

One full cycle equals 360°. Therefore, if 8 divisions correspond to 360°, then:

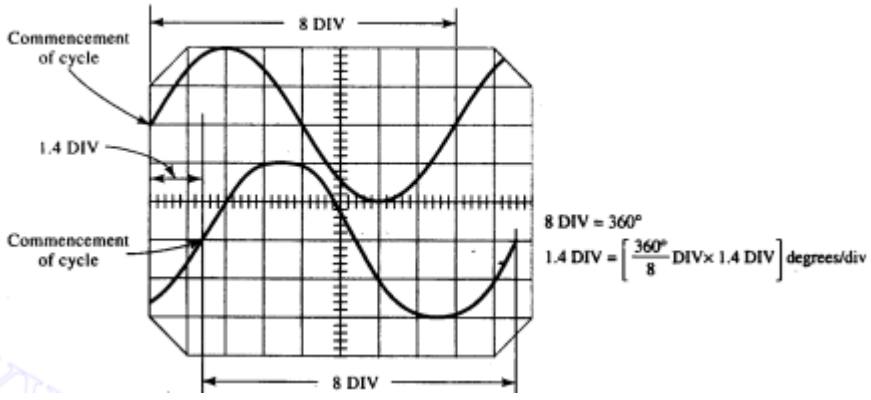


Figure 4-6 phase difference between two waveforms

$$1 \text{ div} = \frac{360^\circ}{8} = 45^\circ$$

Thus, the phase difference is

$$\phi = 1.4 \text{ div} \times (45^\circ/\text{div})$$
$$= 63^\circ$$

ϕ = (phase difference in divisions) × (degrees/div)

Example 4-2

Determine the amplitude, frequency and phase difference between the two waveforms

Solution

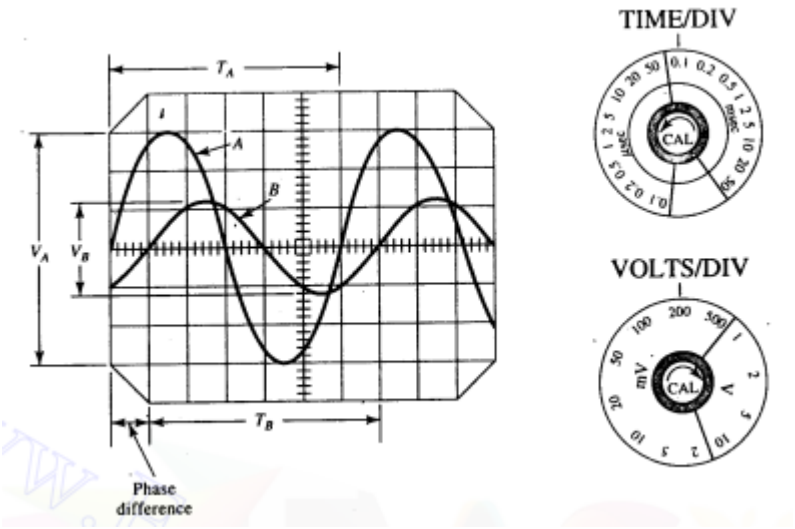
V_A peak-to-peak = (6 vertical div) × (200 mV/div)

= 1.2 V

T_A = (6 horizontal div) × (0.1 ms/div)

= 0.6 ms

$$f_A = \frac{1}{T} = \frac{1}{0.6 \text{ ms}}$$



≈ 1670 Hz

V_B peak-to-peak ≈ (2.4 vertical div) × (200 mV/div)

= 480 mV

T_B = 0.6 ms

$f_B \approx$ 1670 Hz

one cycle = (6 horizontal div) = 360°

phase difference ≈ 1 div

$$= \frac{360^\circ}{6} = 60^\circ$$

4-6 Function Generators

Basic Circuit:

A function generator produces sine, square, and triangular waveforms, and sometimes a ramp waveform. It allows for variable output frequency and amplitude and may include a DC offset adjustment.

A sine/square-wave generator can be used as a function generator by adding a circuit to convert the square wave into a triangular wave. Typically, this involves using an integrator and a Schmitt trigger circuit, as illustrated in Figure 4-7.

The Schmitt trigger circuit is a noninverting type. It works by rapidly switching its output between its maximum positive and negative levels when the input voltage crosses specific trigger points. When the input voltage exceeds the upper trigger point (UTP), the output jumps to the most positive level. Conversely, when the input drops below the lower trigger point (LTP), the output falls to the most negative level.

In this setup, the inverting input terminal of the Schmitt op-amp is grounded. The operational amplifier typically has a high voltage gain, around 200,000, so only a small voltage difference between the inverting and noninverting terminals is needed to drive the output to saturation in either direction.

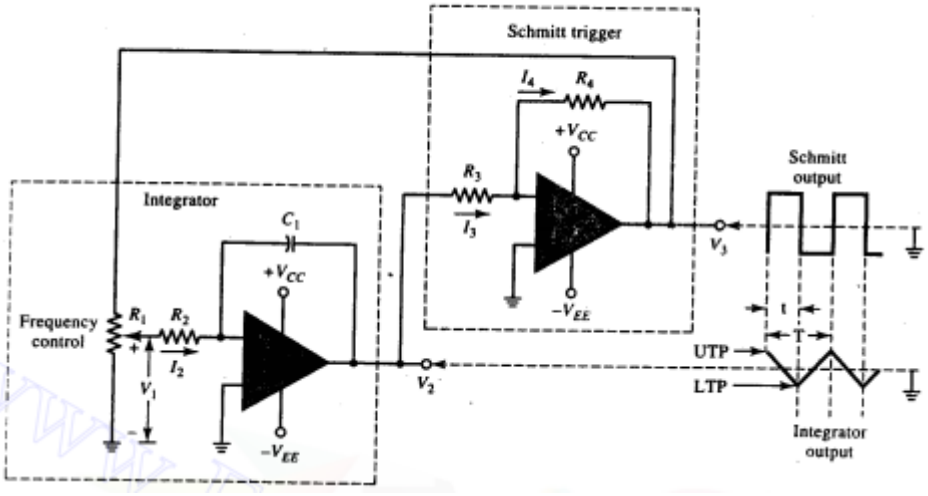


Figure 4-7 basic Function Generator circuit

In this circuit, the noninverting terminal of the op-amp is grounded, while the inverting terminal is connected to the op-amp output through capacitor C_1. If C_1 were replaced with a short circuit, the op-amp would function as a voltage follower, with both the output and inverting input terminals at ground level, since the noninverting input is grounded.

With C_1 in place and no charge initially on C_1, the circuit also behaves as a voltage follower, maintaining all three terminals of the op-amp at ground level.

When C_1 is charged to a voltage of +1 V on the right side and -1 V on the left side, the inverting input remains at ground level, and the output stabilizes at +1 V with respect to ground. Conversely, if C_1 is charged to -1 V on the right and +1 V on the left, the output voltage becomes -1 V with respect to ground.

Now, if a positive input voltage $+V_i$ is applied to R_2, the left terminal of R_2 is at $+V_i$ while the right terminal, which is connected to the grounded noninverting input, remains at ground level. Therefore, the entire voltage V_i appears across R_2, resulting in a constant current I_2 flowing through R_2.

$$I_2 = \frac{V_1}{R_2}$$

The current I_2 is chosen to be much larger than the input bias current of the operational amplifier. As a result, nearly all of I_2 flows into C_1, charging it with a polarity of + on the left and - on the right. As C_1 charges, its voltage increases linearly, causing the op-amp output voltage to decrease linearly. When the polarity of V_i is reversed, I_2 changes direction, and C_1 charges with the opposite polarity, leading to a reversal in the integrator output voltage direction.

In Figure 4-7, the integrator input voltage comes from the Schmitt trigger output, and the integrator output feeds back into the Schmitt circuit. Assume the Schmitt trigger output is +14 V and the integrator output is at ground level. Since V_i is positive, I_2 charges C_1 with + on the left and - on the right, resulting in the integrator output voltage V_2 decreasing linearly from ground level. When V_2 reaches the Schmitt's lower trigger point (LTP), the Schmitt output switches rapidly to -14 V. This reverses V_i, causing I_2 to flow in the opposite direction and C_1 to charge in the reverse polarity. Consequently, V_2 increases linearly from the LTP.

When V_2 reaches the Schmitt's upper trigger point (UTP), the Schmitt output switches back to +14 V. V_i becomes positive again, and I_2 charges C_1 with the opposite polarity, resulting in V_2 decreasing once more. This process repeats, producing a triangular waveform at the integrator output. The Schmitt trigger output is a square wave, positive when the integrator output is decreasing and negative when it is increasing. The frequency of these waveforms depends on the time it takes for C_1 to charge from the UTP to the LTP and vice versa.

$$C = \frac{It}{\Delta V}$$

or

$$t = \frac{C\,\Delta V}{I}$$

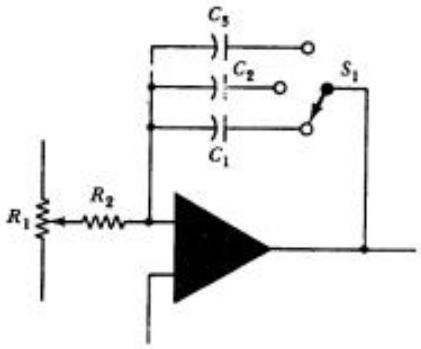


Figure 4-8 output frequency of basic Function Generator

Example 11-3

Consider the integrator circuit shown in Figure 4-7, where

$C_1 = 0.1\ \mu F$, $R_1 = 1\ k\Omega$, and $R_2 = 10\ k\Omega$.

Schmitt trigger circuit has ±3 V trigger points, calculate the output frequency when the moving contact of R_1 is at the top of the potentiometer, and when it is at 10% of R_1 from the bottom. The supply voltage is $V_{CC} = \pm 15$ V.

$$V_3 \simeq \pm(V_{CC} - 1\text{ V}) = \pm(15\text{ V} - 1\text{ V})$$
$$= \pm 14\text{ V}$$

For contact at top of R_1:

$$V_1 = V_3 = 14\text{ V}$$
$$I_2 = \frac{V_1}{R_2} = \frac{14\text{ V}}{10\text{ k}\Omega}$$
$$= 1.4\text{ mA}$$
$$\Delta V = \text{UTP} - \text{LTP} = 3\text{ V} - (-3\text{ V})$$
$$= 6\text{ V}$$

$$t = \frac{C\,\Delta V}{I_2} = \frac{0.1\ \mu\text{F} \times 6\text{ V}}{1.4\text{ mA}}$$
$$= 0.43\text{ ms}$$
$$f = \frac{1}{2t} = \frac{1}{2 \times 0.43\text{ ms}}$$
$$= 1.17\text{ kHz}$$

For R_1 contact at 10% from bottom:

$$V_1 = 10\% \text{ of } V_3 = 10\% \text{ of } 14 \text{ V}$$
$$= 1.4 \text{ V}$$
$$I_2 = \frac{1.4 \text{ V}}{10 \text{ k}\Omega} = 0.14 \text{ mA}$$

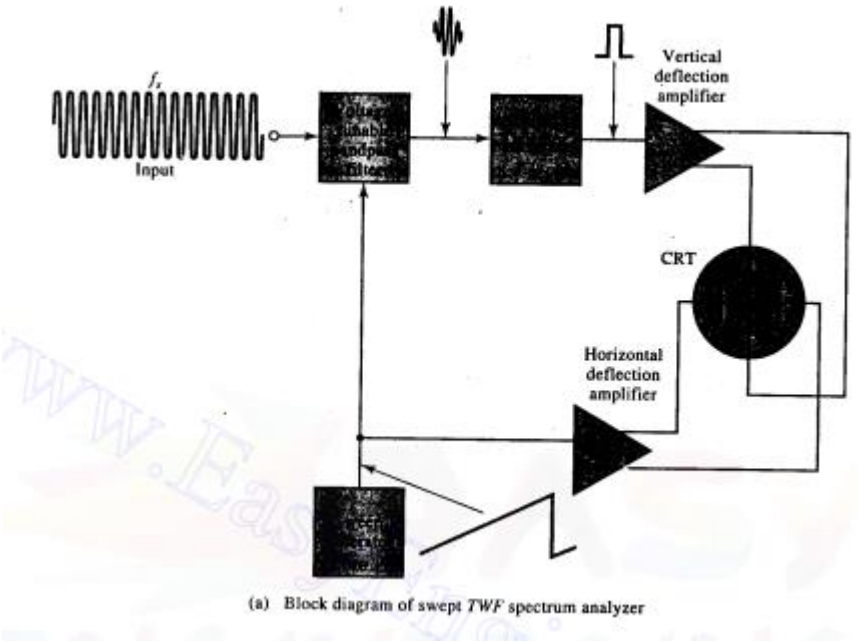
$$t = \frac{0.1 \ \mu\text{F} \times 6 \text{ V}}{0.14 \text{ mA}}$$
$$\approx 4.3 \text{ ms}$$
$$f = \frac{1}{2 \times 4.3 \text{ ms}}$$
$$\approx 117 \text{ Hz}$$

4-7 Swept TRF Spectrum Analyzer

A spectrum analyzer decomposes an AC signal into its frequency components and displays them as vertical lines on a CRT screen. Each vertical line's amplitude reflects the amplitude of a specific frequency component, while its horizontal position indicates the corresponding frequency.

In a swept TRF spectrum analyzer, as shown in Figure 4-9(a), a sweep generator creates a linear ramp voltage that controls both the horizontal sweep of the CRT and the tuning of a voltage-tunable bandpass filter. The filter’s passband frequency sweeps from a minimum f_1 to a maximum f_2 as the ramp voltage. An input signal with frequency f_s passes through the filter only when the filter's passband is tuned to f_s. The signal's amplitude is converted to a DC voltage by a detector and used to modulate the vertical deflection of the CRT, creating a vertical line on the screen.

An input sine wave might appear clean on an oscilloscope but could show harmonic distortion on a spectrum analyzer. While oscilloscopes display signal amplitude versus time (time domain), spectrum analyzers show amplitude versus frequency (frequency domain), revealing harmonic components.



(a) Block diagram of swept *TWF* spectrum analyzer

Figure 4- 9 block diagram of a swept TWF spectrum analyzer